Fundamental Sciences of Reliability

Mohammad Modarres
Center for Risk and Reliability

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Acknowledgments

The Team:

1. Dr. Anahita Imanian (Former PhD Student)
2. Dr. Victor Ontiveros (Former PhD Student)
3. Dr. Huisung Yun (Former PhD candidate)
4. Dr. Foad Karimian (Former PhD Student)
5. Dr. Mehdi Amiri (Former Postdoc)
6. Dr. Ali Kahirdeh (Former Postdoc)
7. Ms. Christine Sauerbrunn (Former MS Student)
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Is Reliability Engineering a discipline of Engineering, Statistics, or Operations Research?

What is Engineering?

Webster Dictionary: The **application of science and mathematics** by which the properties of matter and the sources of energy in nature are made useful to people

What is Statistics?

The discipline that concerns "the collection, organization, analysis, interpretation, and presentation of masses of numerical data". It is a sub-field of mathematics

What is Operations Research?

The discipline that deals with the development and application of advanced analytical methods to improve decision-making.

If Reliability Engineering is a legitimate engineering discipline, what fundamental sciences underpinning its theories and methods?

- Reliability Engineering is formally defined as applying *scientific know-how* to a component, product, plant, or process in order to ensure that it performs its intended function, *without failure*, for the required time duration in a specified environment.
- Statistics as a branch of mathematical sciences is an undisputable foundation of reliability engineering.
- What is the corresponding physical science?
- Since " failure" in engineering systems occur because of damage, then we need to identify the physical science that describes *material damage.*

Thermodynamics as the Underlying Physical Science of Reliability Engineering

- Describe materials cumulative damage and overstress damage within the irreversible nonequilibrium thermodynamics, statistical mechanics and information theory frameworks.
- Understand coupled damage/failure phenomena such as corrosion-fatigue
- Enhance engineering applications, such as Prognosis and Health Management (PHM) of structures.
- Applications of thermodynamics in reliability engineering is not new, but my research focuses on addressing the questions posed in the previous slides through the nonequilibrium thermodynamics, statistical mechanics and information theory.

A Thermodynamic Interpretation of Materials Damage

Rudlof Clausius

Second Law of Thermodynamics: In an isolated system, entropy will always increase until it reaches a maximum value.

Ludwig Boltzmann

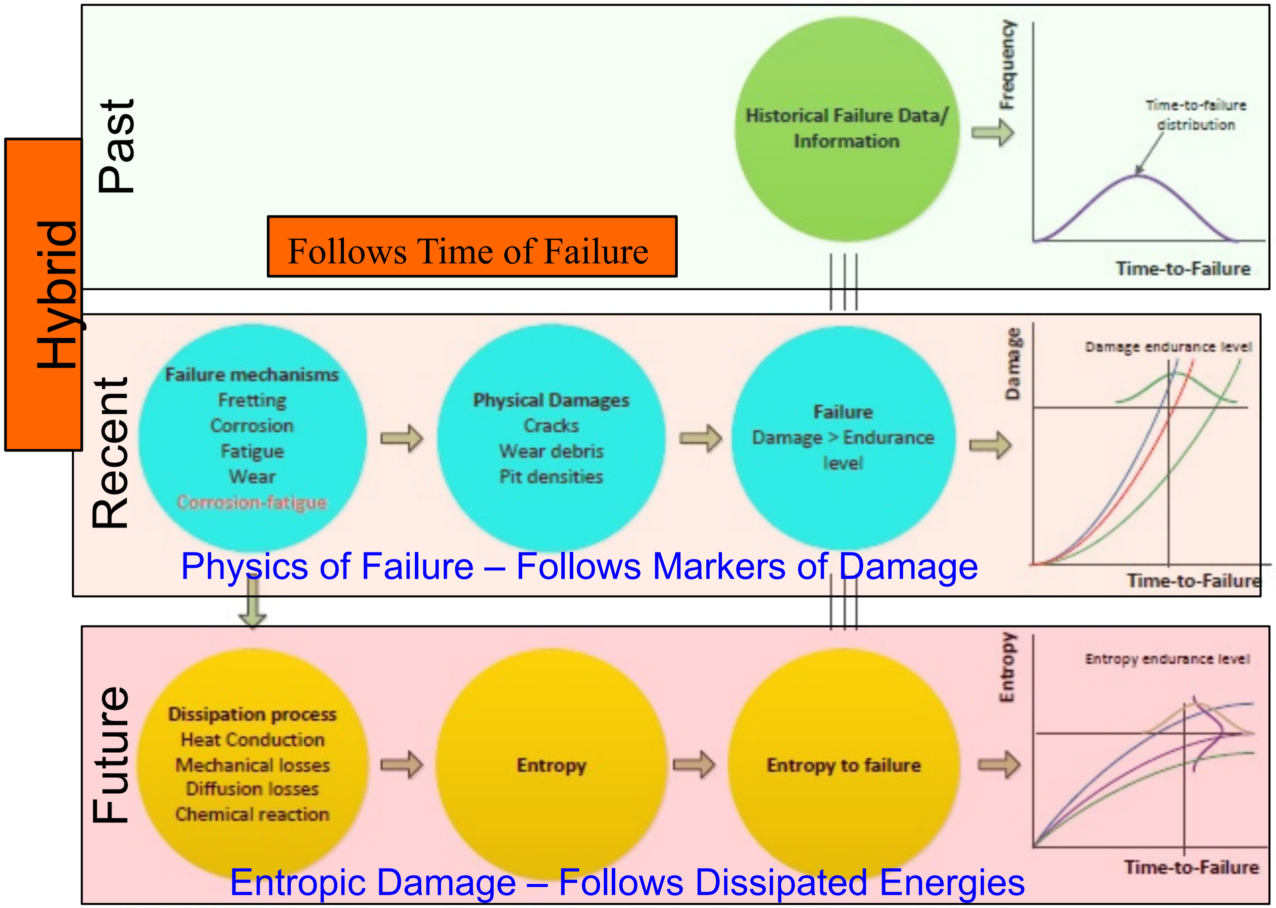
Second Law of Thermodynamics (Statistical Mechanics Version): In an isolated system, the system will always progress to a macrostate that corresponds to the maximum number of microstates.

Interpretation of Material Damages in terms of the 2nd Law

All damages resulting from failure mechanisms or overstress failures share a common feature: Dissipation of Energy.

Dissipation: a fundamental determinant of irreversibility, can be described well within the context of non-equilibrium thermodynamics.

Thermodynamics as the Science of Reliability



Why Entropy?

- ✓ Entropy is independent of the path to failure ending at similar total entropy at failure
- ✓ Entropy accounts for complex synergistic effects of interacting failure mechanisms
- ✓ Entropy is scale independent

An Entropic Theory of Damage

Degradation mechanisms → Damage → Dissipation energies → Entropy generation

Damage $\equiv$ Entropy

An entropic theory follows[1]:

Failure occurs when the accumulated total entropy generated exceeds the entropic-endurance of the unit

- Entropic-endurance describes the capacity of the unit to withstand entropy
- Entropic-endurance of identical units is equal
- Entropic-endurance of different units is different
- Entropic-endurance to failure can be measured (experimentally) and involves stochastic variability

- In this context we define Damage as normalized Entropy: $D = \frac{s_d - s_{d_0}}{s_{d_E} - s_{d_0}}$

Total entropy generation, s_d, monotonically increases starting at time zero from a theoretical value of zero or practically some initial entropy, s_{d0}, to an entropic-endurance value, s_d

[1] Anahita Imanian and Mohammad Modarres, A Thermodynamic Entropy Approach to Reliability Assessment with Application to Corrosion Fatigue, Entropy 17.10 (2015): 6995-7020

Total Entropy Generated

- Entropy generation σ involves a thermodynamic force, X_i, and an entropy flux, J_i as:

$$\sigma = \Sigma_{i,j} X_i J_i(X_j) \; ; \quad (i, j=1,\ldots, n)$$

For near equilibrium condition interactions between multiple dissipation processes is captured by the Onsagar reciprocal relations define forces and fluxes. $J_i = \sum_j L_{ij} X_j$ *For example, for Fatigue (f) and Corrosion (c)*

$$J_c = L_{cc}X_c + L_{fc}X_f \text{ and } J_f = L_{cf}X_c + L_{ff}X_f$$

$[L_{ij}]$= Onsager matrix of phenomenological coefficients

- Entropy generation of important dissipation phenomena leading to damage:

$$\sigma = \overbrace{\frac{1}{T^2}\boldsymbol{J}_q.\nabla T}^{\text{Thermal energy}} + \overbrace{\Sigma_{k=1}^{n}\boldsymbol{J}_k\left(\nabla\frac{\mu_k}{T}\right)}^{\text{Diffusion energy}} + \overbrace{\frac{1}{T}\boldsymbol{\tau}:\dot{\boldsymbol{\epsilon}}_p}^{\text{Plastic deformation energy}} + \underbrace{\frac{1}{T}\Sigma_{j=1}^{r}v_j\, A_j}_{\text{Chemical reaction energy}} + \underbrace{\frac{1}{T}\Sigma_{m=1}^{h}c_m\boldsymbol{J}_m(-\nabla\psi)}_{\text{External fields energy}}$$

$\boldsymbol{J}_n$ $(n = q, k, and\ m)$ = thermodynamic fluxes due to heat conduction, diffusion and external fields, T=temperature, μ_k = chemical potential, v_i =chemical reaction rate, $\boldsymbol{\tau}$ =stress tensor, $\dot{\boldsymbol{\epsilon}}_p$ =the plastic strain rate, A_j =the chemical affinity or chemical reaction potential difference, ψ =potential of the external field, and c_m =coupling constant [*, **]

[*] D. Kondepudi and I. Prigogine, *"Modern Thermodynamics: From Heat Engines to Dissipative Structures,"* Wiley, England, 1998.

[**] J. Lemaitre and J. L. Chaboche, "*Mechanics of Solid Materials,*" 3rd edition; Cambridge University Press: Cambridge, UK, 2000.

Examples of Force and Flux of Dissipative Processes

$$\sigma = \Sigma_{i,j} X_i J_i(X_j)\ ; \quad (i, j=1,\ldots, n)$$

Primary mechanism	Thermodynamic force, X	Thermodynamic flow, J	Examples of materials damage process
Heat conduction	Temperature gradient, $\nabla(1/T)$	Heat flux, $\boldsymbol{q}$	Fatigue, creep, wear
Plastic deformation of solids	Stress, $\boldsymbol{\sigma}/T$	Plastic strain, $\dot{\boldsymbol{\varepsilon}}_p$	Fatigue, creep, wear
Chemical reaction	Reaction affinity, A_k/T	Reaction rate, v_k	Corrosion, wear
Mass diffusion	Chemical potential, $-\nabla(\mu_k/T)$	Diffusion flux, $\boldsymbol{J}_k$	Wear, creep
Electrochemical reaction	Electrochemical potential, $\tilde{A}/T$	Current density, i_{corr}/z	Corrosion
Irradiation	Particle flux density, A_r/T	Velocity of target atoms after collision, $\dot{v}_r$	Irradiation damage
Annihilation of lattice sites	Creep driving force $(\tilde{\boldsymbol{\sigma}} - \omega \boldsymbol{I})/T$	Creep deformation rate, $\boldsymbol{R}$	Creep

Table From: Amiri, M. and Modarres, M., *An Entropy-Based Damage Characterization*, Entropy, 16, 2014.

Entropic Approaches: Fatigue Mechanism

Measurements

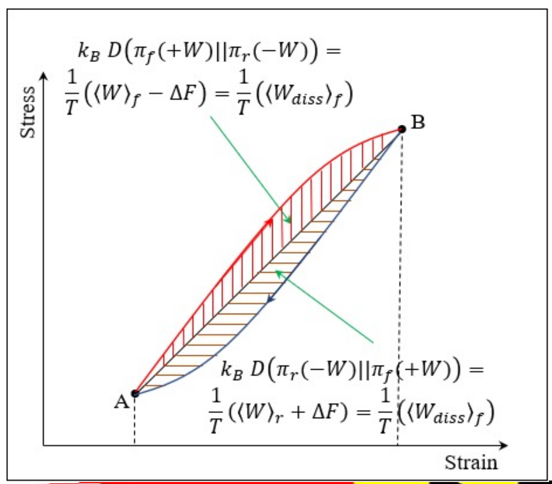
- Dissipative energy
  - Strain energy — Load cell, extensometer
  - Heat (Temperature) — Thermocouple
- Acoustic emission — AE sensors

Entropic Approaches

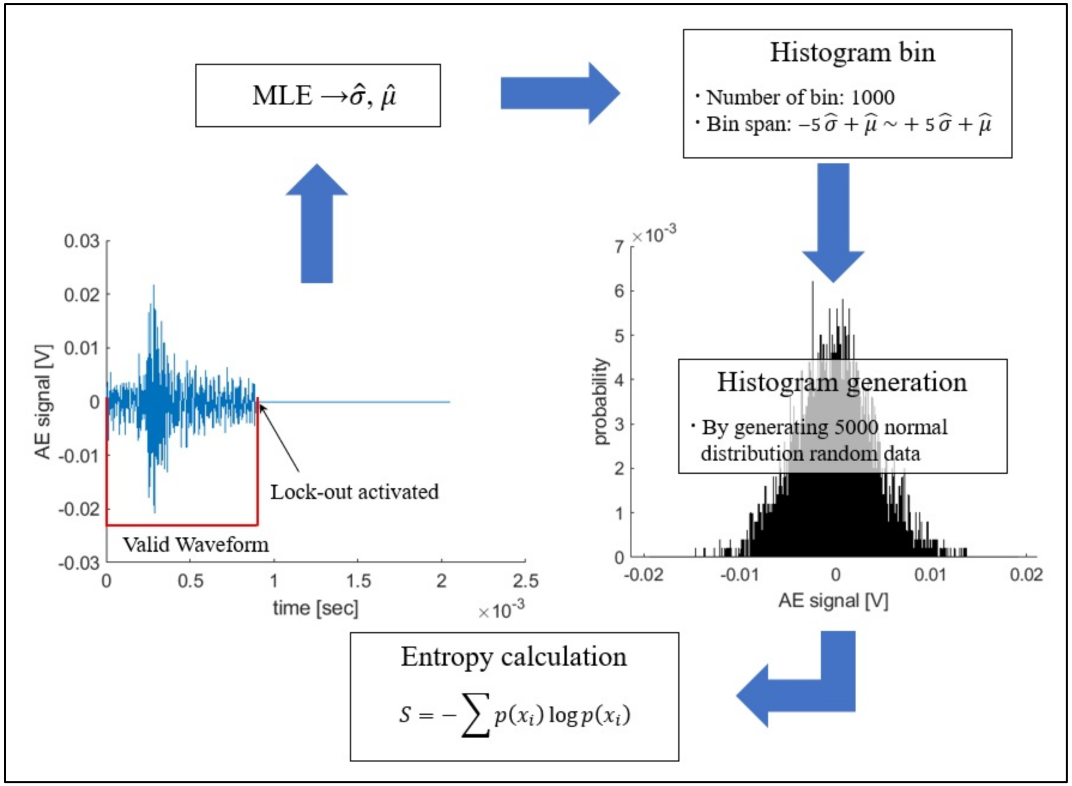
- Thermodynamic entropy: $\frac{1}{T}\tau : \varepsilon_p$
- Jeffreys divergence: $\int \pi_f \ln\left(\frac{\pi_f}{\pi_r}\right)$
- AE entropy: $-\sum p \log p$

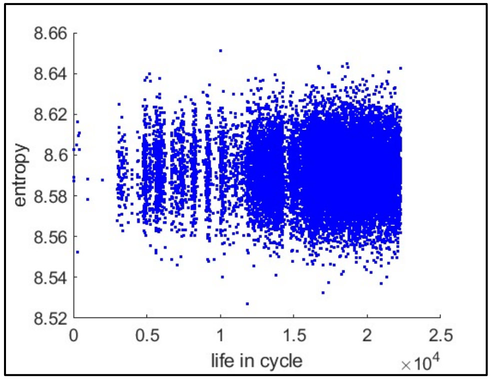
Assessment

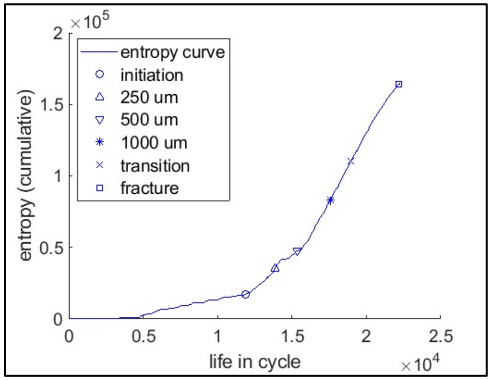
- Thermodynamic entropy:
  - Entropic endurance
  - Damage assessment
  - Life estimation
- Jeffreys divergence:
  - Damage damage
  - Correlation to thermodynamic entropy (k_{pB})
- AE entropy:
  - Damage assessment
  - Compare to AE features



Information Theory Based Entropy



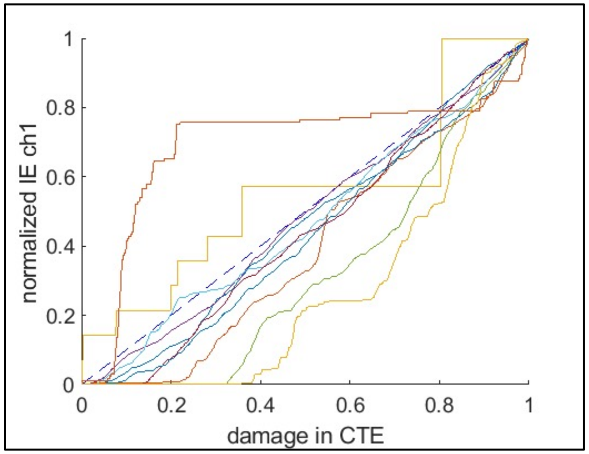




Information Entropy

Normalized damage

$$D = \frac{M_i - M_o}{M_f - M_o}$$



AE information entropy

Thermodynamic Entropy Generation in CF

- Contribution from corrosion activation over-potential, diffusion over-potential, corrosion reaction chemical potential, plastic and elastic deformation and hydrogen embrittlement to the rate of entropy generation:

Electrochemical dissipations

$$\sigma = \frac{1}{T}\left(\boldsymbol{J}_{M,a} z_M F E_{M_{act,a}} + \boldsymbol{J}_{M,c} z_M F E_{M_{act,c}} + \boldsymbol{J}_{O,a} z_O F E_{O_{act,a}} + \boldsymbol{J}_{O,c} z_O F E_{O_{act,c}}\right)$$

$$+\frac{1}{T}\left(\boldsymbol{J}_{M,c} z_M F E_{M_{conc,c}} + z_O F \boldsymbol{J}_{O,c} E_{O_{conc,c}}\right)$$

Diffusion dissipations

$$+\frac{1}{T}\left(\boldsymbol{J}_{M,a}\alpha_M A_M + J_{M,c}(1-\alpha_M)A_M + \boldsymbol{J}_{O,a}\alpha_O A_O + \boldsymbol{J}_{M,a}(1-\alpha_O)A_O\right)$$

Chemical reaction dissipations

$$+\frac{1}{T}\dot{\boldsymbol{\epsilon}}_p : \boldsymbol{\tau} + \frac{1}{T} Y \dot{\boldsymbol{D}}$$

Mechanical dissipations

$$+\sigma_H$$

Hydrogen embrittlement dissipation

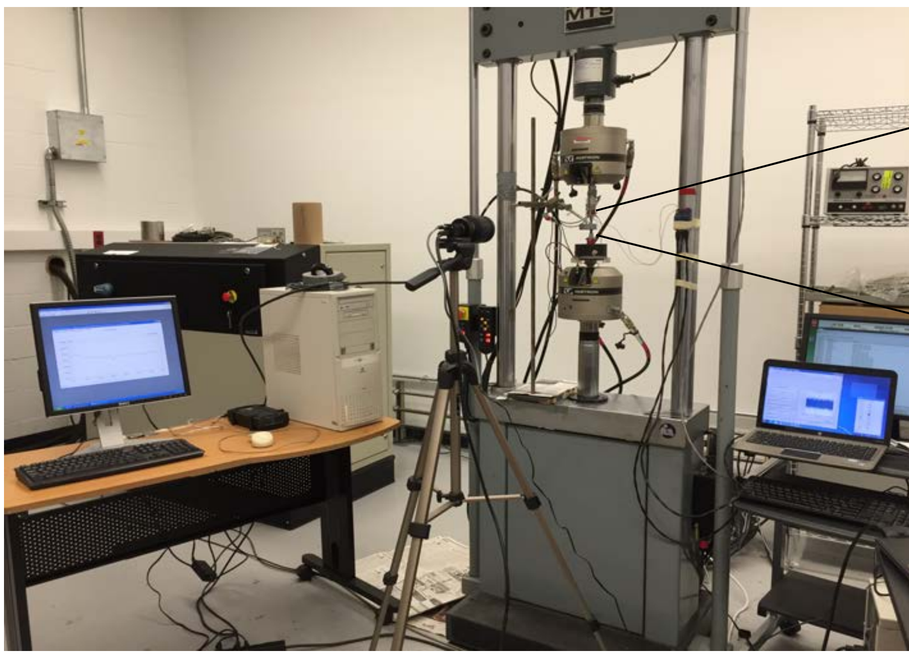
T = temperature, z_M =number of moles of electrons exchanged in the oxidation process, F =Farady number, $J_{M,a}$ and $J_{M,c}$ = irreversible anodic and cathodic activation currents for oxidation reaction, $J_{O,a}$ and $J_{O,c}$ =anodic and cathodic activation currents for reduction reaction, $E_{M_{act,a}}$ and $E_{M_{act,c}}$ =anodic and cathodic over-potentials for oxidation reaction, $E_{O_{act,a}}$ and $E_{O_{act,c}}$ =anodic and cathodic over-potentials for reduction reaction, $E_{M_{conc,c}}$ and $E_{O_{conc,c}}$ =concentration over-potentials for the cathodic oxidation and cathodic reduction reactions, α_M and α_O =charge transport coefficient for the oxidation and reduction reactions, A_M and A_O = chemical affinity for the oxidation and reductions, $\dot{\epsilon}_p$ =plastic deformation rate, τ =plastic stress, $\dot{D}$ =dimensionless damage flux, Y the elastic energy, and σ_H =entropy generation due to hydrogen embrittlement.

Imanian, A. and Modarres. M, "*A Thermodynamic Entropy Based Approach for Prognosis and Health Management with Application to Corrosion-Fatigue,*" 2015 IEEE International Conference on Prognostics and Health Management, 22-25 June 2015, Austin, USA.

Corrosion Fatigue (CF) Experimental Set up

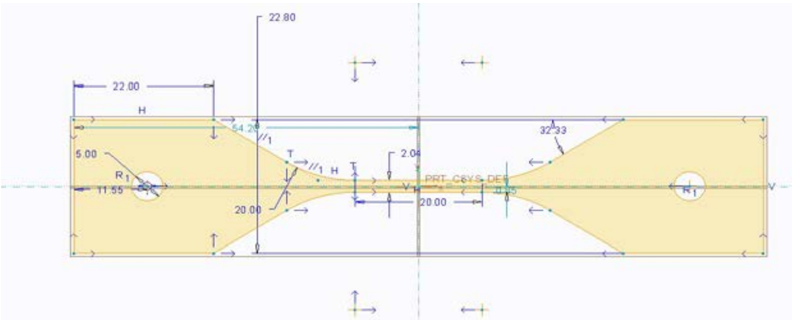
- Fatigue tests of Al 7075-T651 in 3.5% wt. NaCl aqueous solution acidified with a 1 molar solution of HCl, with the pH of about 3.5, under axial load controlled and free corrosion potential
- Specimen electrochemically monitored via a Gamry potentiostat using Ag/AgCl reference electrode maintained at a constant distance (2 mm) from the specimen, a platinum counter electrode, and the specimen as the working electrode
- Digital image correlation (DIC) technique used to measure strain

CF tests done while measuring the open circuit potential (OCP) vs. reference electrode during load-unload



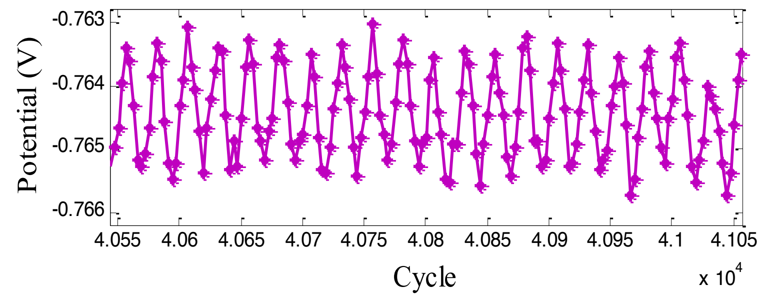
Electrochemical corrosion cell made of plexiglass

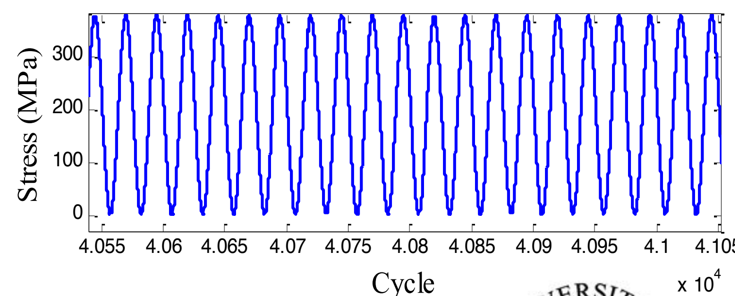
CF Test Procedures



Forces and fluxes were measured under CF

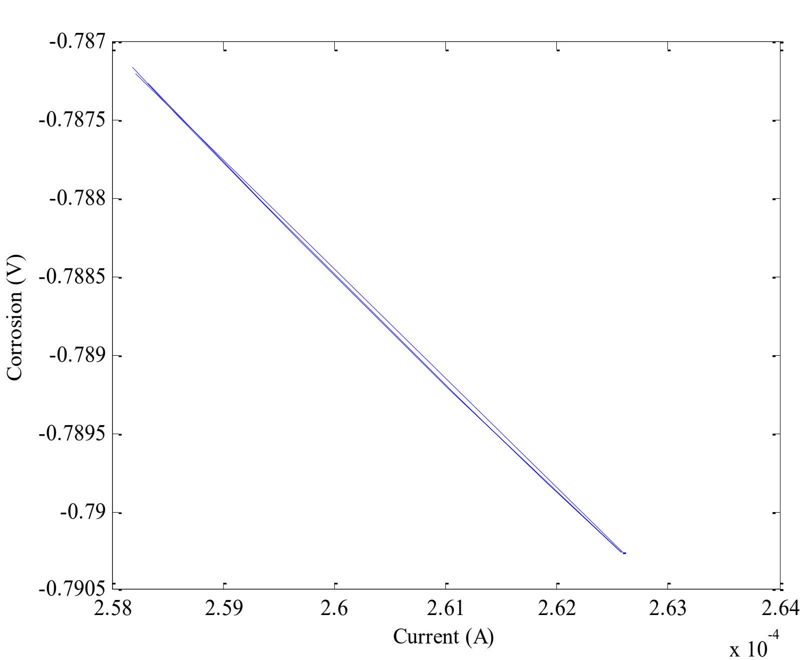
- Performed CF tests for 16 samples at 87%, 80%, 70% and 57% of yield stress (460 MPa), load ratio = 0.1, loading frequency=0.04Hz
- Tests stopped after failure of specimens

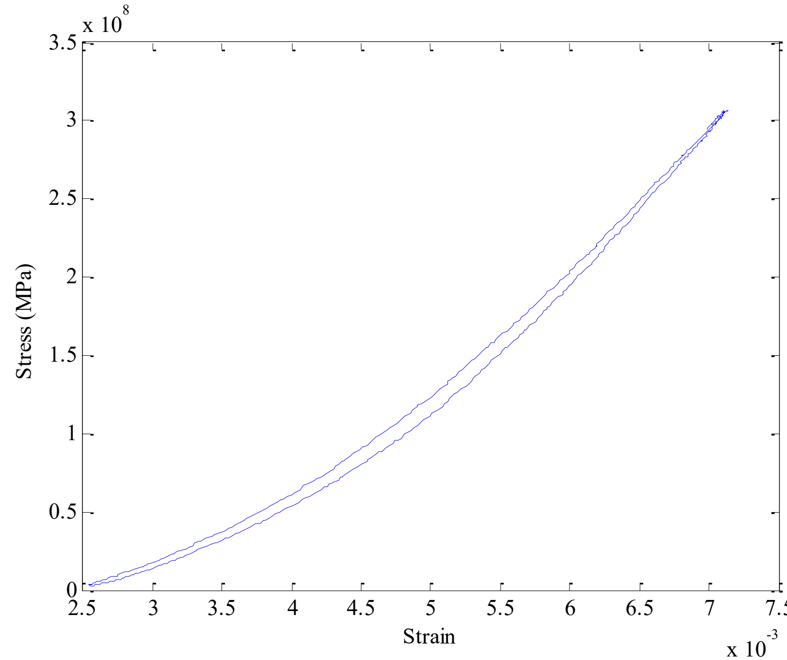




Entropy Generation in CF

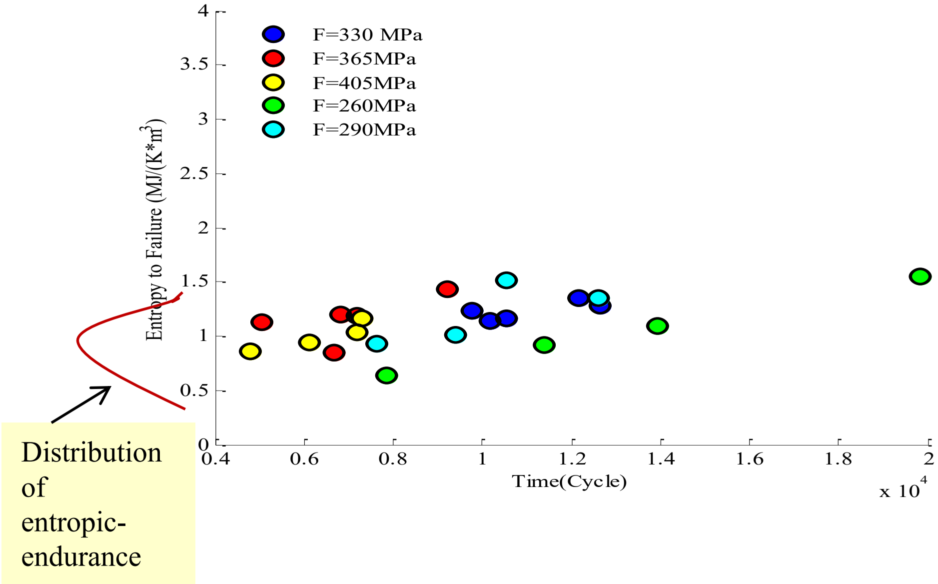
- Total entropy is measured from the hysteresis loops resulted from fatigue (stress-strain) and corrosion (potential-electrical) in each loading cycle

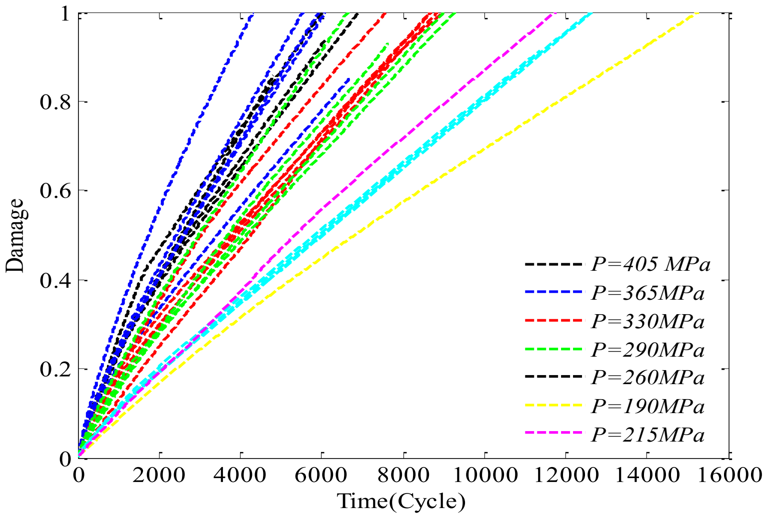




Entropic Endurance and Entropy-to-Failure

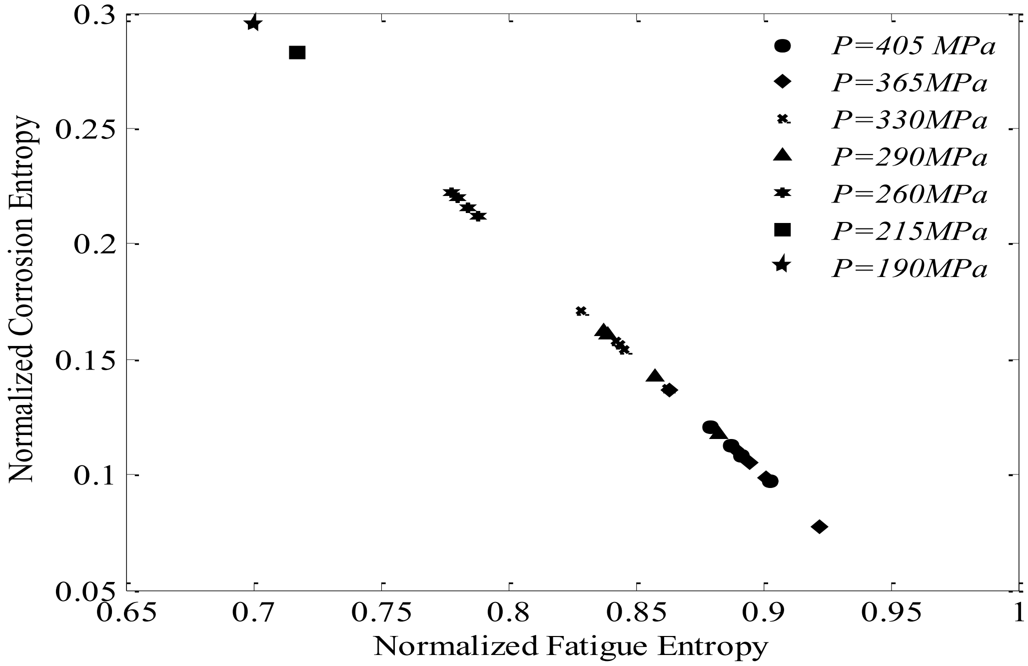
- Similarity of the total entropy-to-failure for all tests supports the entropic theory of damage proposed
- More tests needed to reduce the epistemic uncertainties and further confirm the theory





Ratio of Corrosion and Fatigue Entropies to the Total Entropy

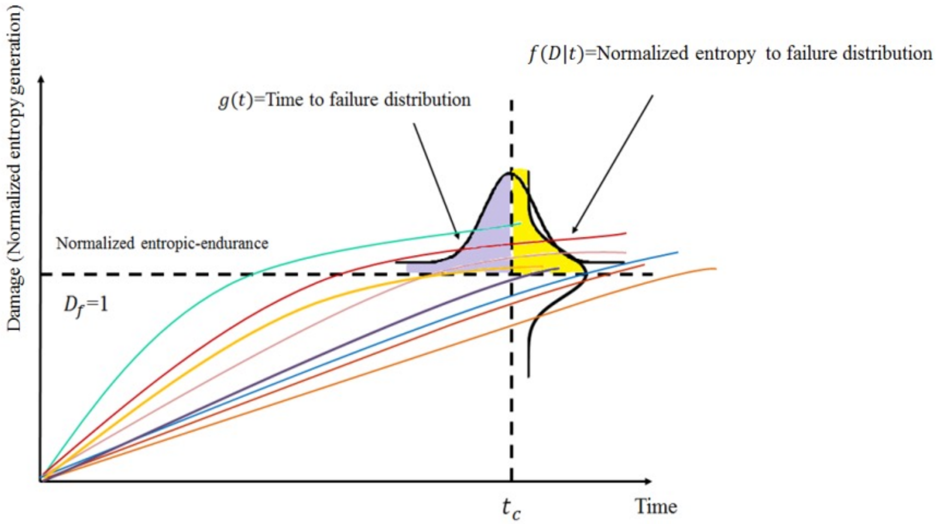
- Reducing fatigue stress allows more time for corrosion



Thermodynamics of Damage: A Reliability Perspective

- Materials, environmental, operational and other types of variabilities in degradation forces impose uncertainties on the total entropic damage

- Assuming a constant *entropic-endurance*, D_f



- The reliability function can be expressed as [1]

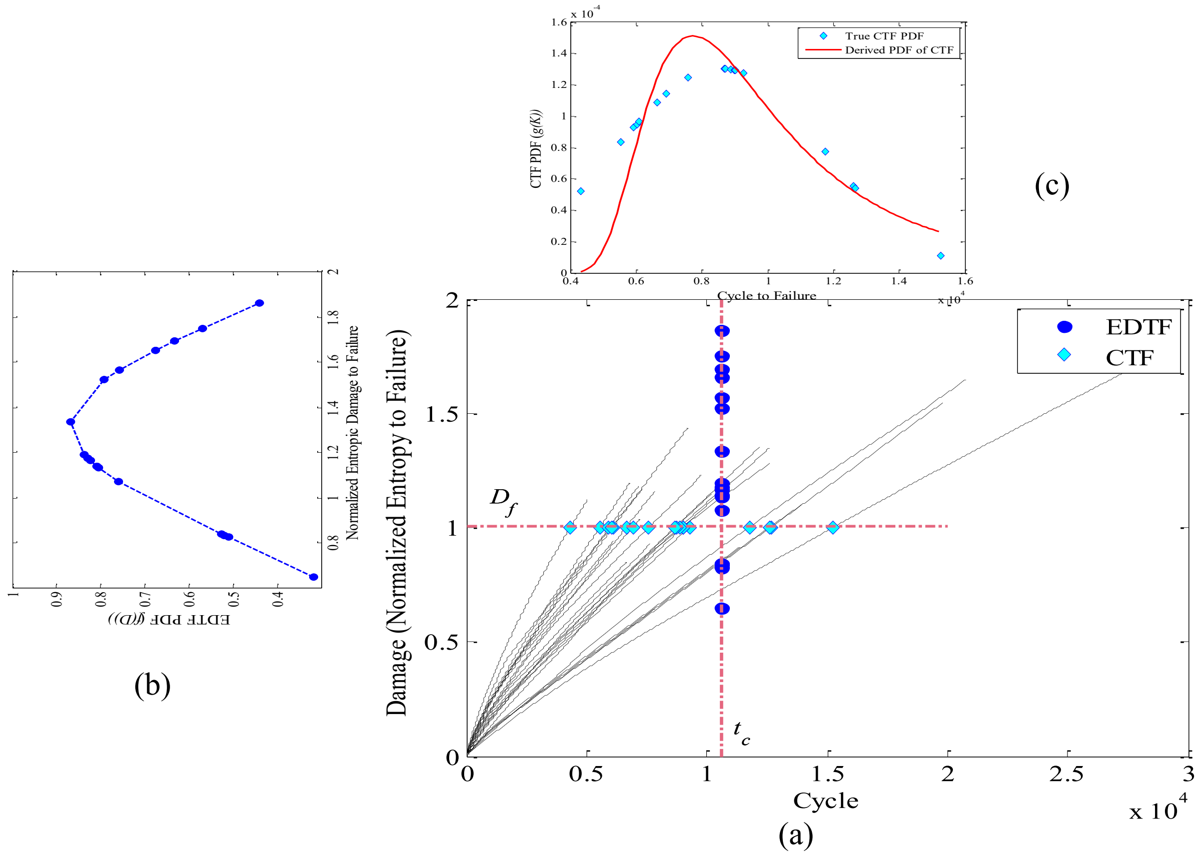
$$P_r(T \leq t_c) = \int_0^{t_c} g(t)dt = 1 - \int_0^{D_f=1} f(D)dD$$

$$R(t_c) = 1 - P_r(T \leq t_c) = \int_0^{D_f=1} f(D)dD$$

T_c =*Current operating time;* $g(t)$=*distribution of time-to-failure,* $f(D|t)$= *distribution of damage at t*

[1] Thermodynamics as a Fundamental Science of Reliability, A. Imanian, M. Modarres, Int. J. of Risk and Reliability, Vol.230(6), pp.598-608. DOI: 10.1177/1748006X16679578.(2016).

CF Life Derived From a Science-based Reliability Theorem



Conclusions

- Reliability engineering (time-to-failure) can be directly *described and derived* withing the laws of nonequilibrium Thermodynamics, Statistical Mechanics and and information theory.
- Entropy correlates with damage that ultimately causes a failure
- Fundamental science supporting reliability engineering exists
- This theory of damage allows for incorporation of all common and interacting dissipative processes in reliability engineering
- Experimental entropic-based reliability results have provided a proof this claim
- More research is prudent

Thank you