Characterization and Quantification of Uncertainties in Probabilistic Fracture Mechanics with Applications to Probability of Detection and Sizing of Flaws and Cracks

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Presentation at the 3rd International Seminar on Probabilistic Methods for Nuclear Applications

22-24 October 2019, Rockville, Maryland, USA

Topics Covered: Back to Basics!

- Definition of Probability and Uncertainty
- Interpretation of Uncertainty: Frequentist vs. Bayesian
- Uncertainty Representations
- Examples in the Context of Probability of Detection (POD) and Sizing Flaws
- Conclusion

Definition of Probability: Frequentist or Classical Viewpoint

- Frequentist Probability: the relative number of occurrences of an event in a large number of *identical and independent* trials (i.e., assumes iid datasets).
- Works well for an ensemble of repeated events, such as tossing a coin or failures of identical equipment, over many iid trials.
- What about probability of unrepeatable events? For example, existence of intelligent life on another planet; or, rare and unidentical events (e.g., SGTR)
- Where do we find an ensemble of those planets or steam generators?

Definition of Probability: Subjectivist or Bayesian Viewpoint

- Subjectivist Probability: the degree of belief (or certainty) of an individual in the truth of a proposition.
- Ideally rational individuals give the same subjective prior probabilities, considering the same information.
- Data and information (evidence) objectively combined into posterior probabilities to continuously suppress the subjectivity of the prior.

Confidence Intervals: Classical Uncertainty Interpretation in Estimation

- Frequentist confidence interval treats unknown parameters $\boldsymbol{\theta}$ as fixed, and the ensemble (sample of the data) as random.
- Interprets the confidence interval as a statistic of the sample.
- For example with 95% confidence a frequentist expects 95% of the confidence intervals of the iid redrawn samples of the data would contain the true mean value of θ.
- Considering a set of detected flaws, the frequentist says that if for example you generate 99 more iid samples, 95 of those samples will contain the true mean flaw size, and 5 won't.

Credibility Intervals: Bayesian Uncertainty Interpretation in Estimation

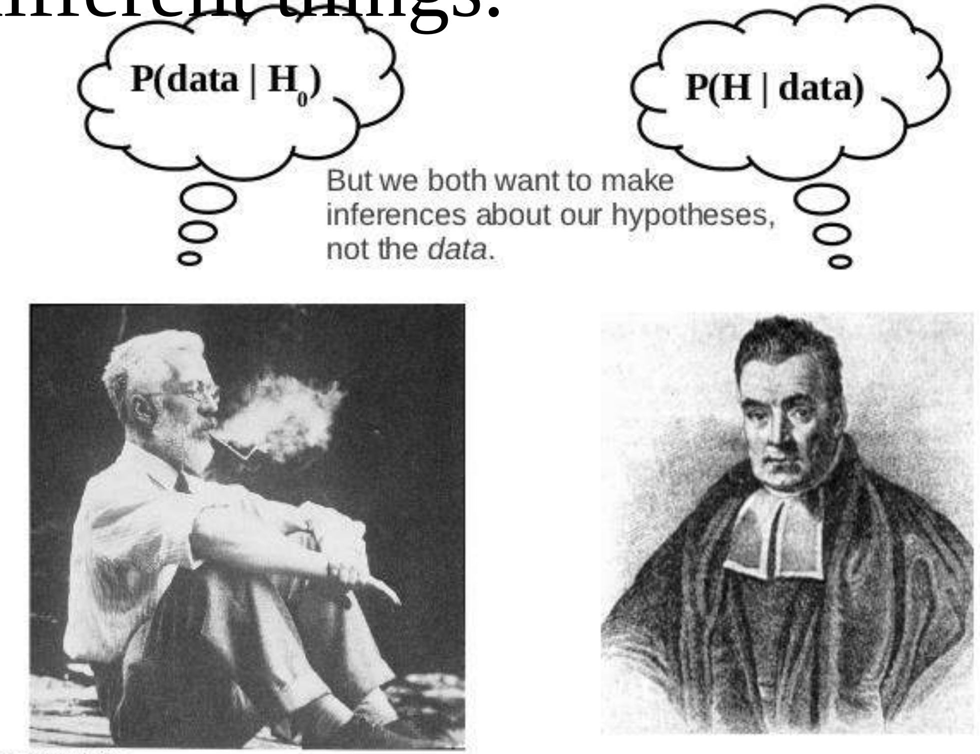
- Bayesian interval (probability bounds) treat parameters $\boldsymbol{\theta}$ as random and the data as fixed, and creates marginal distributions of $\boldsymbol{\theta}$ given the dataset (or broadly the evidence).
- For example, the 95% Bayesian interval is interpreted as the probability (i.e., 95%) that the true value of θ falls within the interval, given the dataset (evidence).
- Considering the detected flaws and other related information, Rev. Bayes' says: the likelihood that the interval contains the true model parameter is 95%.
- Numerically speaking: the credibility interval (bound) corresponds exactly to the classical confidence interval, if the prior probability is entirely "uninformative", albeit with different interpretations.

Confidence vs. Credibility Intervals

- Let a 95% confidence interval (classical) for the mean flaw size given a random sample be (2.69, 6.04) mm:
  - This does not say that 95% of flaws in the sample fall between 2.69 and 6.04 mm!
  - Rather we are 95% confident that the average flaw size is between 2.69 and 6.04 mm.
- Bayesian interpretation is more natural because we want to make a probability statement regarding the range of the true flaw sizes.

Confidence vs. Credibility Intervals (Cont.)

- Confidence and credibility intervals in your uncertainty estimations are close, but mean different things.

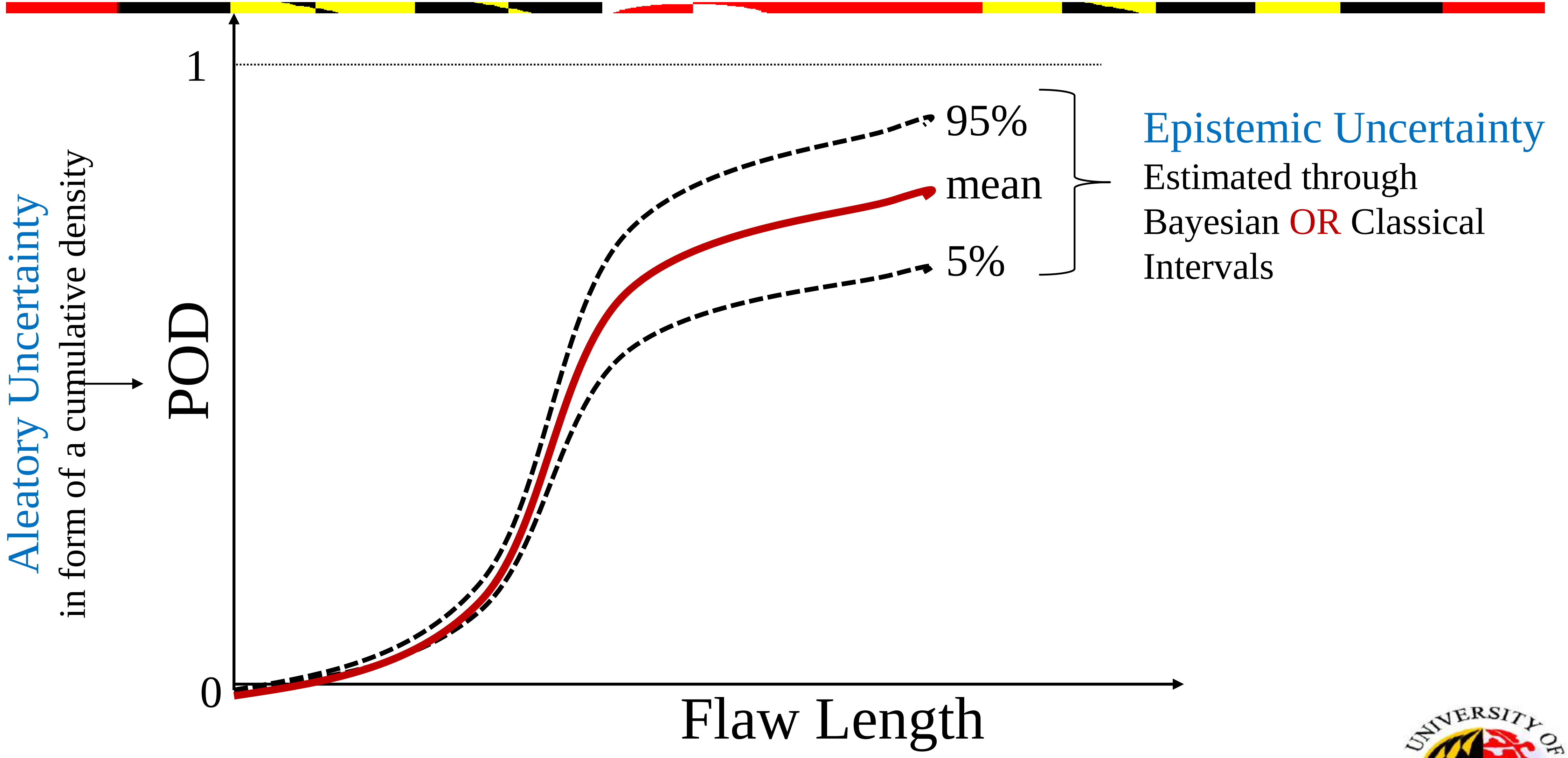


Corey Chivers, 2012

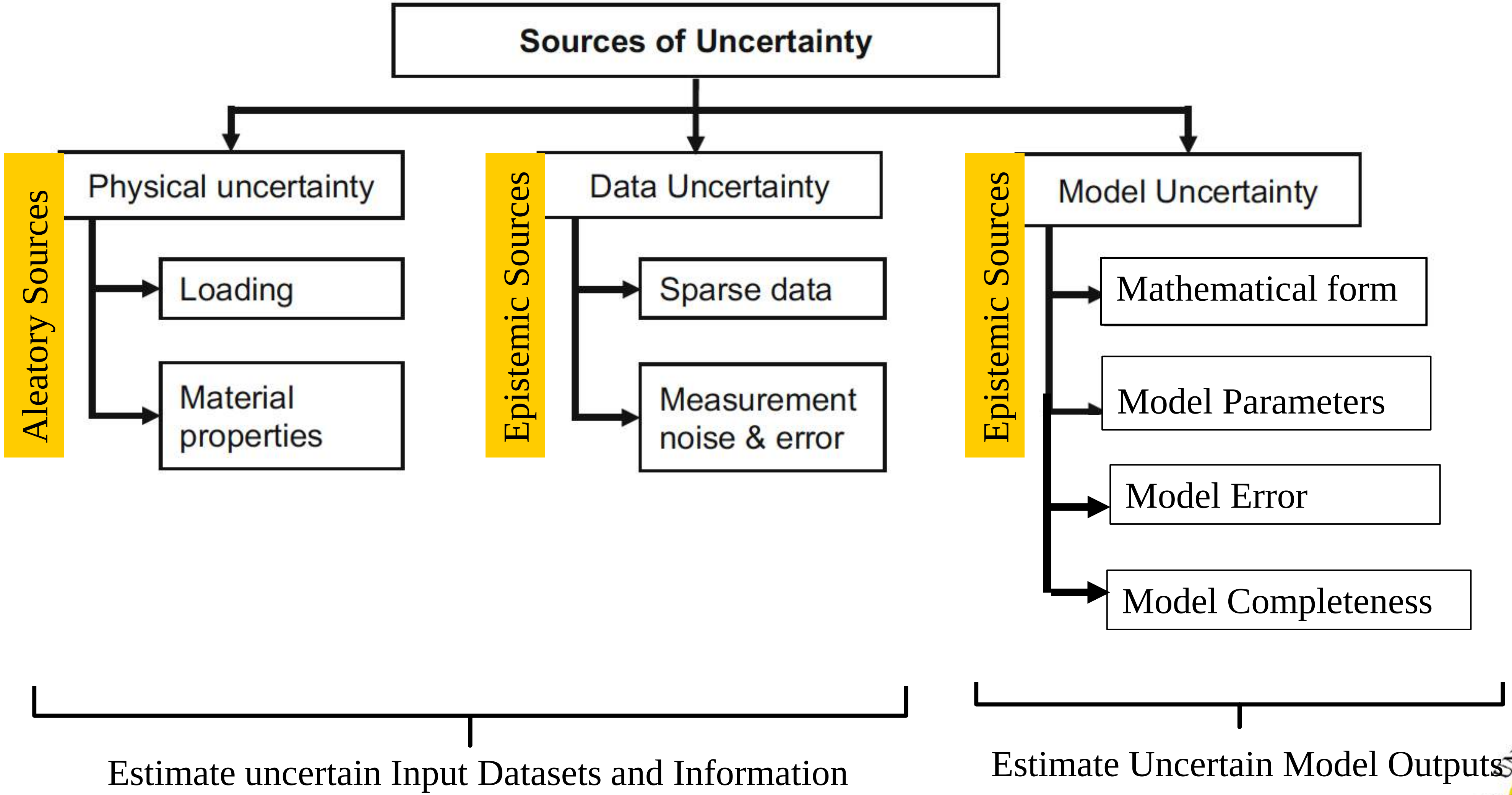
Types of Uncertainty (Bayesian Viewpoints)

- All uncertainties are *epistemic* (driven by lack of data and information)!
- Some are irreducible despite observation of new information and data; we call them *aleatory.*
- Epistemic uncertainties are reducible with new information and data.

Uncertainties in Probability of Detection Model



Sources of Uncertainties in PFM

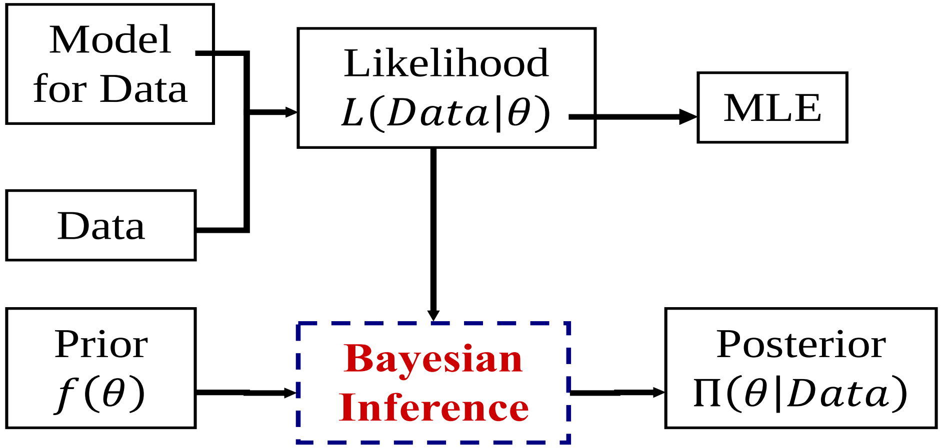


Uncertainty Estimation: MLE and Bayesian

- Data: $D = \{a_1, a_2 \ldots a_n\}$
- Distribution Model: $f(a; \boldsymbol{w})$
- Prior: subjective distribution of parameters $\boldsymbol{w}$
- Bayes' inference: posterior distribution of $\boldsymbol{w}$:

$$p(\boldsymbol{w}|\mathrm{D}) = \frac{L(D|\boldsymbol{w})p(\boldsymbol{w})}{p(D)}$$

- Likelihood: $L(D|\boldsymbol{w})$ describes how probable is the observed dataset (evidence) for various values of $\boldsymbol{w}$
- Model prediction: $f(a) = \int \ldots \int f(a; \boldsymbol{w})p(\boldsymbol{w}|\mathrm{D})d\,\boldsymbol{w}$



Three Probabilistic Model Development Types

Type I: *Probabilistic model with uncertain parameter*: for example $\mathrm{f}(x|\vec{w})$ to represent flaw size distribution, where $\vec{w}$ is the vector of model parameters, which by itself is modeled as a multivariate distribution in Bayesian estimation to account for epistemic uncertainties.
For example prior of $\vec{w}$: $g(\vec{w})\sim MVNPDF(\mathrm{M}_{\vec{w}}, \Sigma_{\vec{w}})$

Type 2: $Probabilistic\ model\ with\ constant\ parameters\ and\ uncertain\ model\ error.$
$\mathrm{f}(x|w_{mean}) + \varepsilon(0, \sigma_x)$. The error term is modeled by a distribution with a mean zero and a positive variance, $g(\sigma_x)$ that represents the epistemic uncertainties..

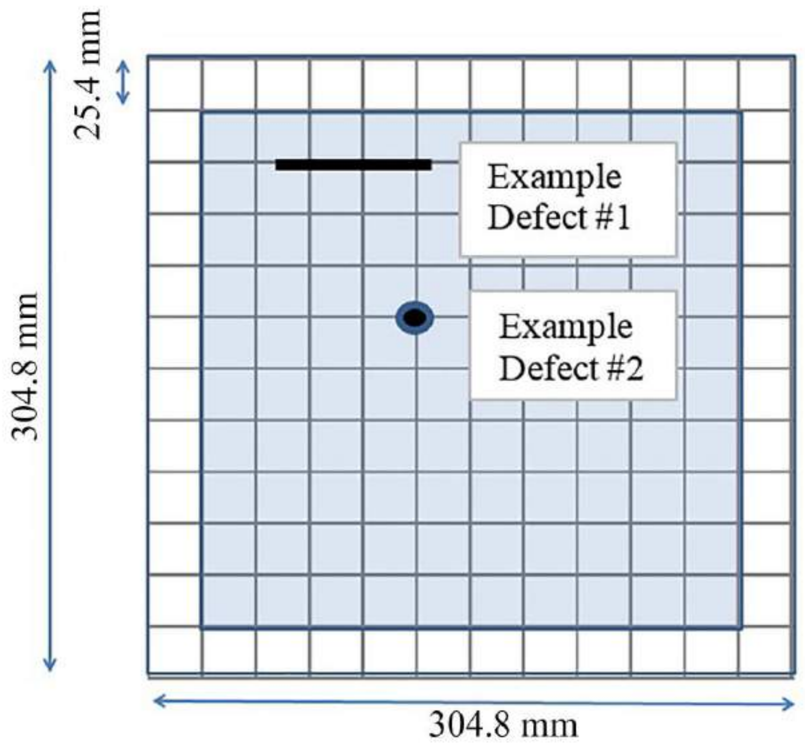
Type 3: *Probabilistic model with parameter uncertainties and uncertain model error.*
$\mathrm{f}(x|\vec{w})$ + $\varepsilon(0, \sigma_x)$. The prior multivariate distribution $g(\vec{w}, \sigma_x)$ represent uncertainties

The multivariate distributions are either estimated using MLE or Bayesian

POD: Definition of Data and Terms

- Flaw Sizing Classification

Flaw Size	Description	89 Detected Flaws
Small	$a < 2.54$ mm	17
Medium	$2.54 \leq a < 25.4$ mm	21
Large	$a \geq 25.4$ mm	51

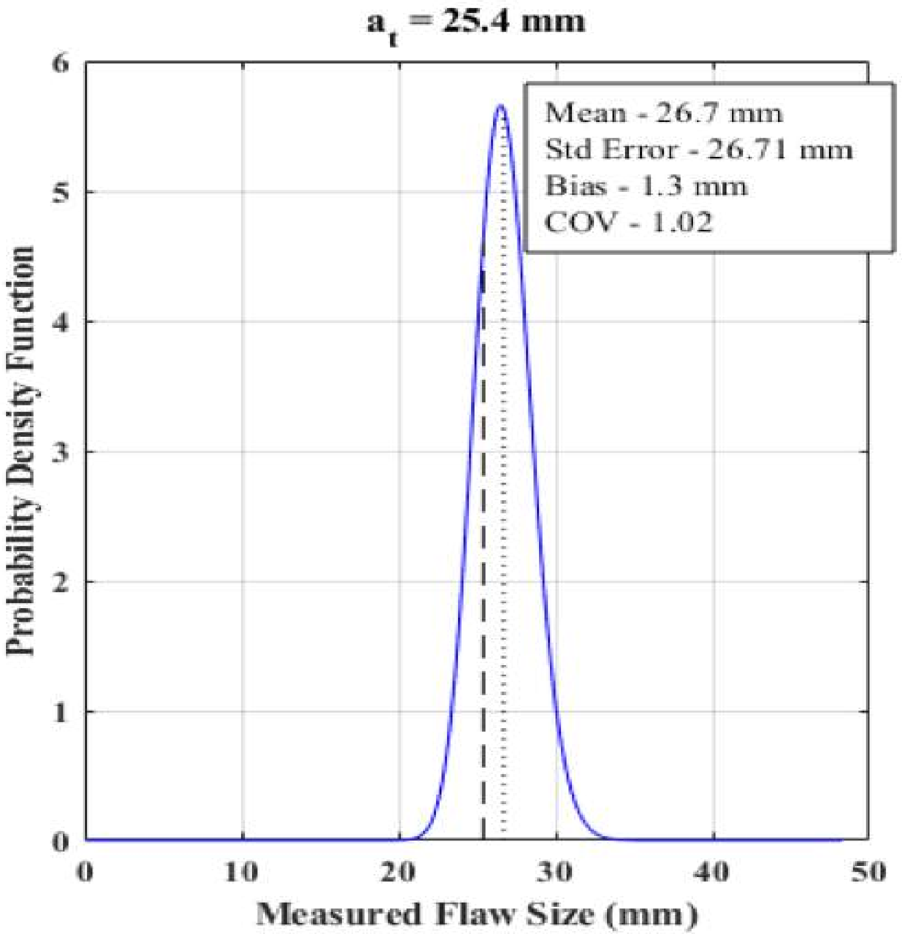


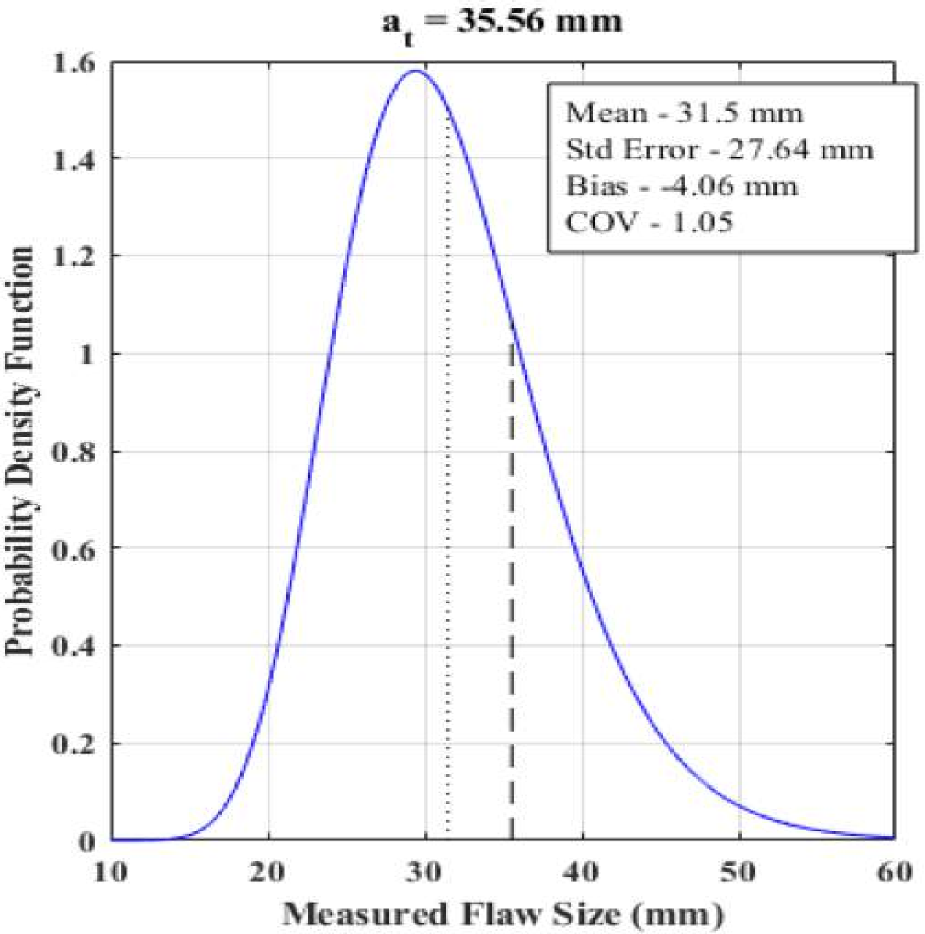
A diagram of a test panel with two types/sizes of defects
(Barrett, Smith, & Modarres 2018)

Partial List of the Detection and Sizing Data

Test No.	Measured (in)	True (in)	Detected
19	2.9	2.8	1
20	0.3	2.25	1
21	4.3	4.25	1
22	0.2	0.03	1
23	0	0.03	0
24	0	0.03	0
25	0.2	0.03	1
26	1	1	1
27	0	0.03	0
28	3.25	3.15	1
29	0.5	0.125	1
30	4	4	1
31	1.5	1.4	1
32	0.25	0.25	1
33	3	3	1

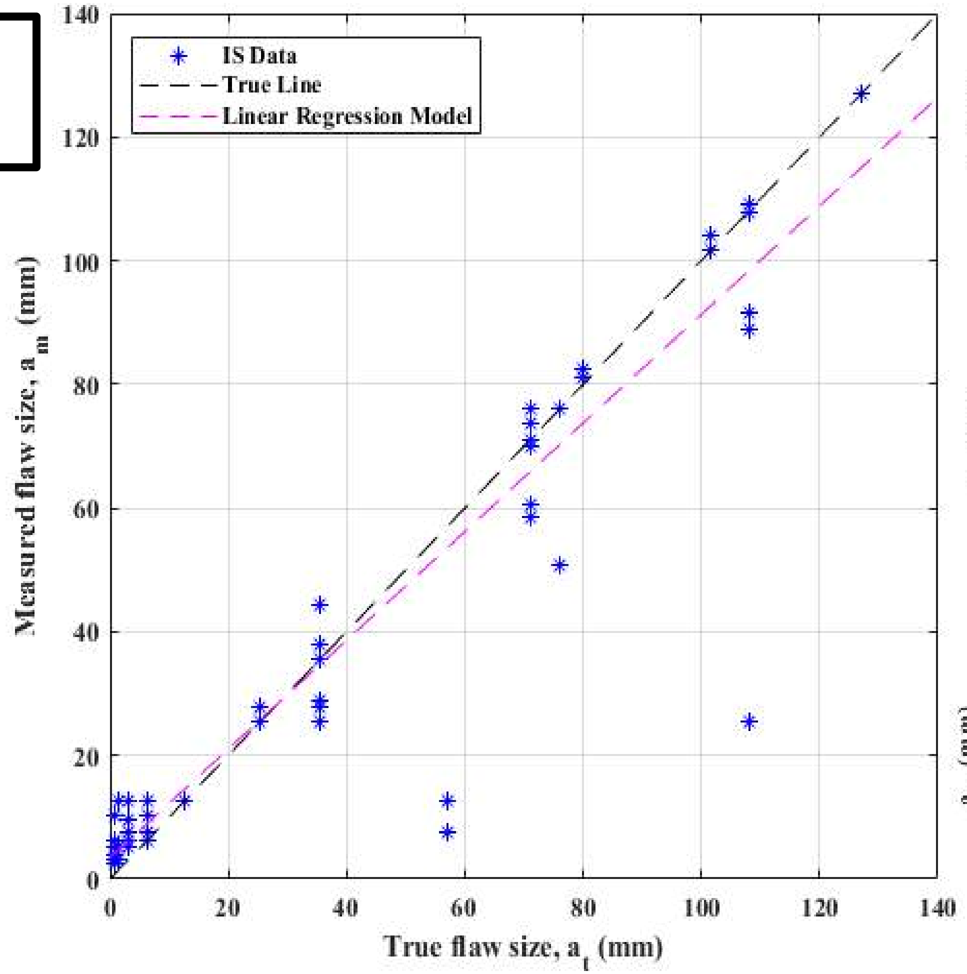
Distribution of Measured Flaw Sizes

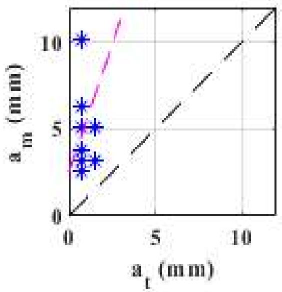




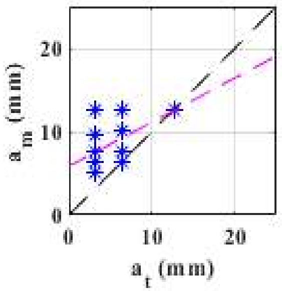
Data Variability Flaw Sizing

Full Group:
$a_m = 0.88a_t + 3.44$

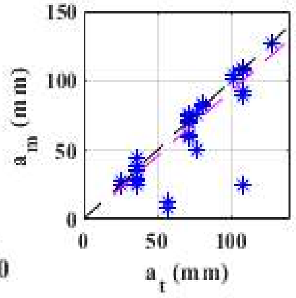




Small Flaw Group:
$a_m = 2.92a_t + 2.54$



Medium Flaw Group:
$a_m = 0.53a_t + 5.86$



Large Flaw Group:
$a_m = 0.94a_t - 1.42$

Bayesian Estimation: Likelihood Definition for POD

- The likelihood for a POD is based on the Bernoulli distribution and given as,

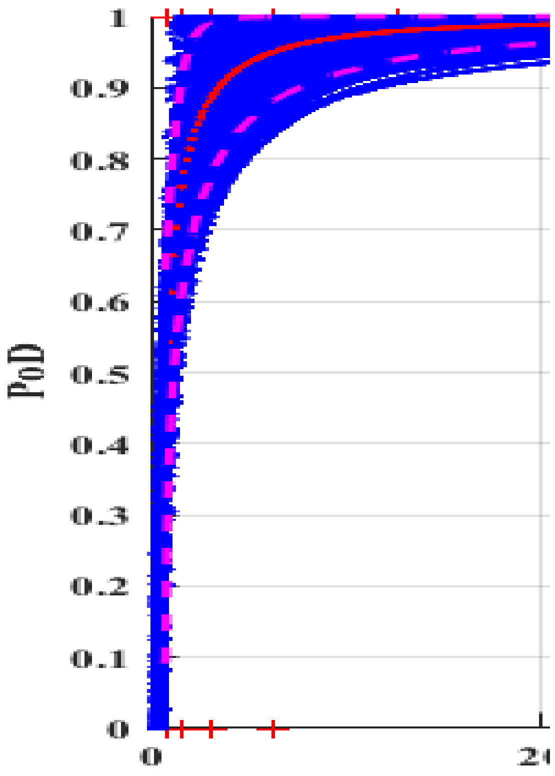
$$l = \prod_{i=1}^{D} PoD(a_i|\vec{w}) \prod_{j=1}^{ND} \left[1 - PoD\left(a_j|\vec{w}\right)\right]$$

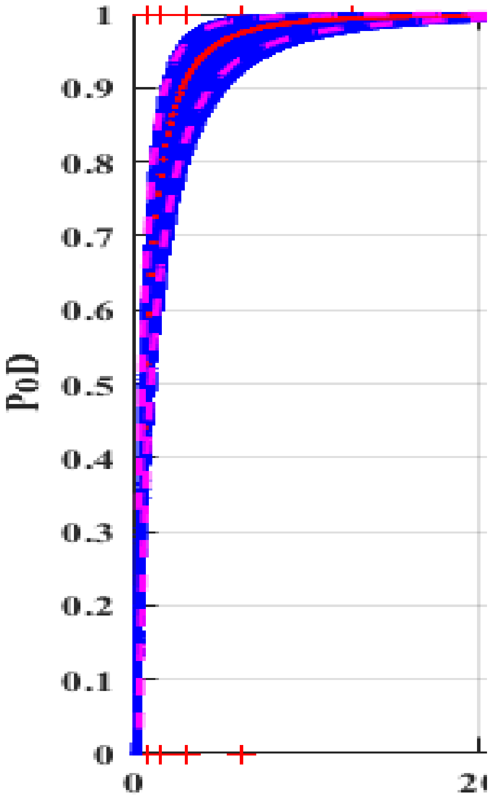
- where D is the total detections and ND is the total non-detections.

POD Uncertainty: MLE vs. Bayesian Estimates

- **Lognormal**

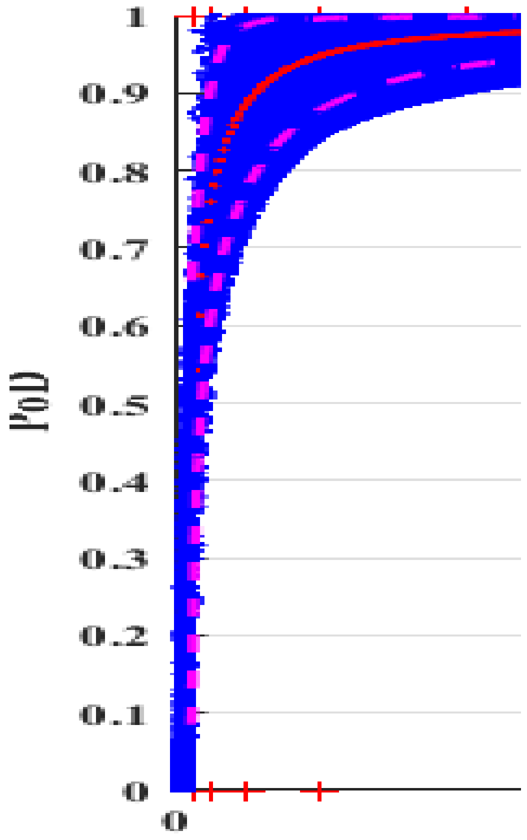
Type I Model

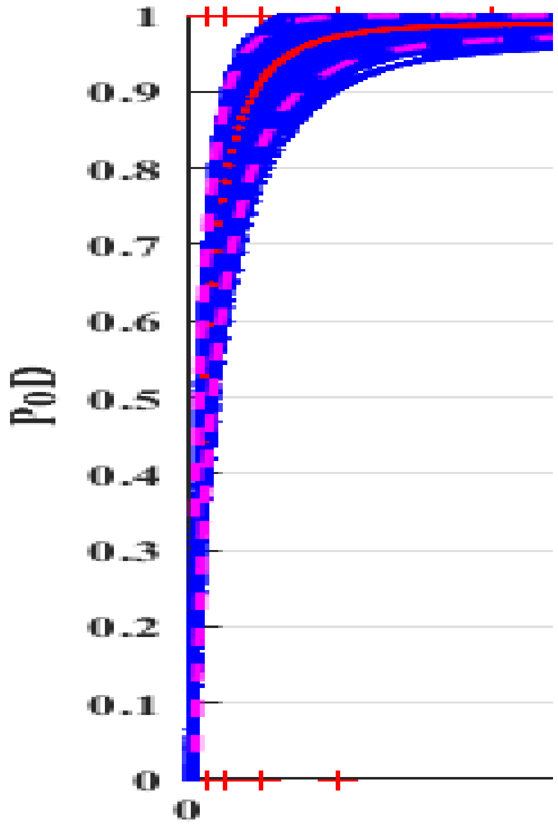




POD Uncertainty: MLE vs. Bayesian Estimates

- **<u>Lognormal</u>**
- **Type III**:
- Model &
- Parameter





Conclusion

- Mix-up in the definition of probability and interpretation of uncertainty in the frequentist vs. Bayesian estimation in engineering applications persists
- More work in this area is warranted, if PFM is to be extensively used in regulatory and safety improvement arenas
- PFM analysts should better communicate the meaning of their results to stakeholders for better acceptance and credibility

Thank you