Small Crack Fatigue Growth and Detection Modeling with Uncertainty and Acoustic Emission Application

Presentation at the

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Outline

- Overview of Fatigue Crack and Acoustic Emission
- Experimental overview and examples of results
- Probabilistic Modeling
- Results
- Conclusions

Traditional Crack Growth/Detection Modeling

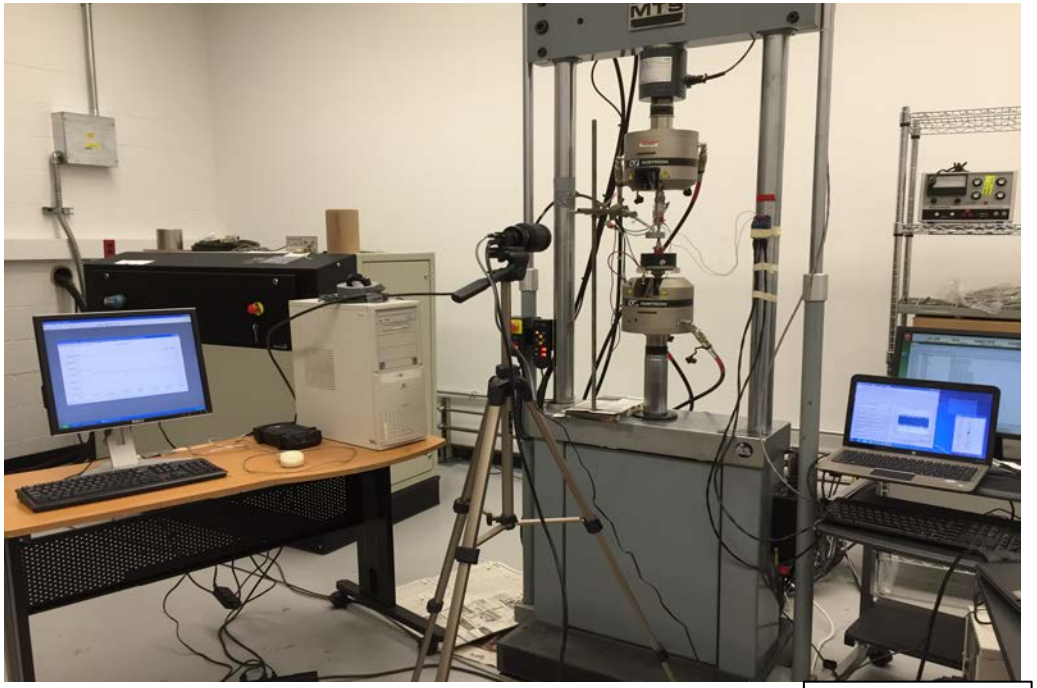
- Traditional empirical modeling of crack damage is typically used to model direct observable crack initiation and growth

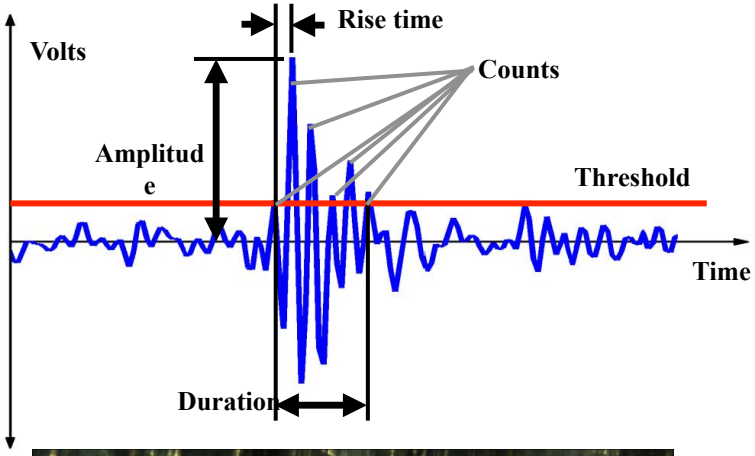
- Recent advances in crack growth allows use of indirect precursors of crack such as acoustic emission (AE) for online monitoring

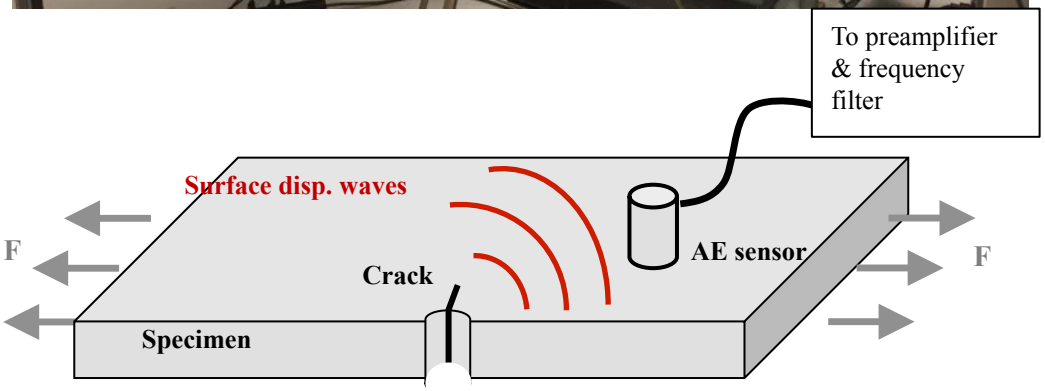
- Wide variety of crack growth and detection models, however these empirical models contain uncertainties:
  1. Model/Data Uncertainty
  2. Physical Variability
  3. Model Error

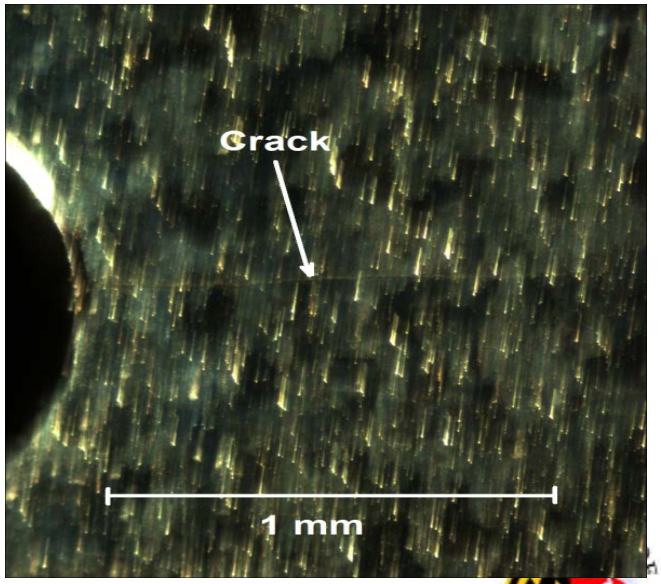
- Identification of these uncertainties are necessary to improve crack growth/detection models

Fatigue Testing with AE



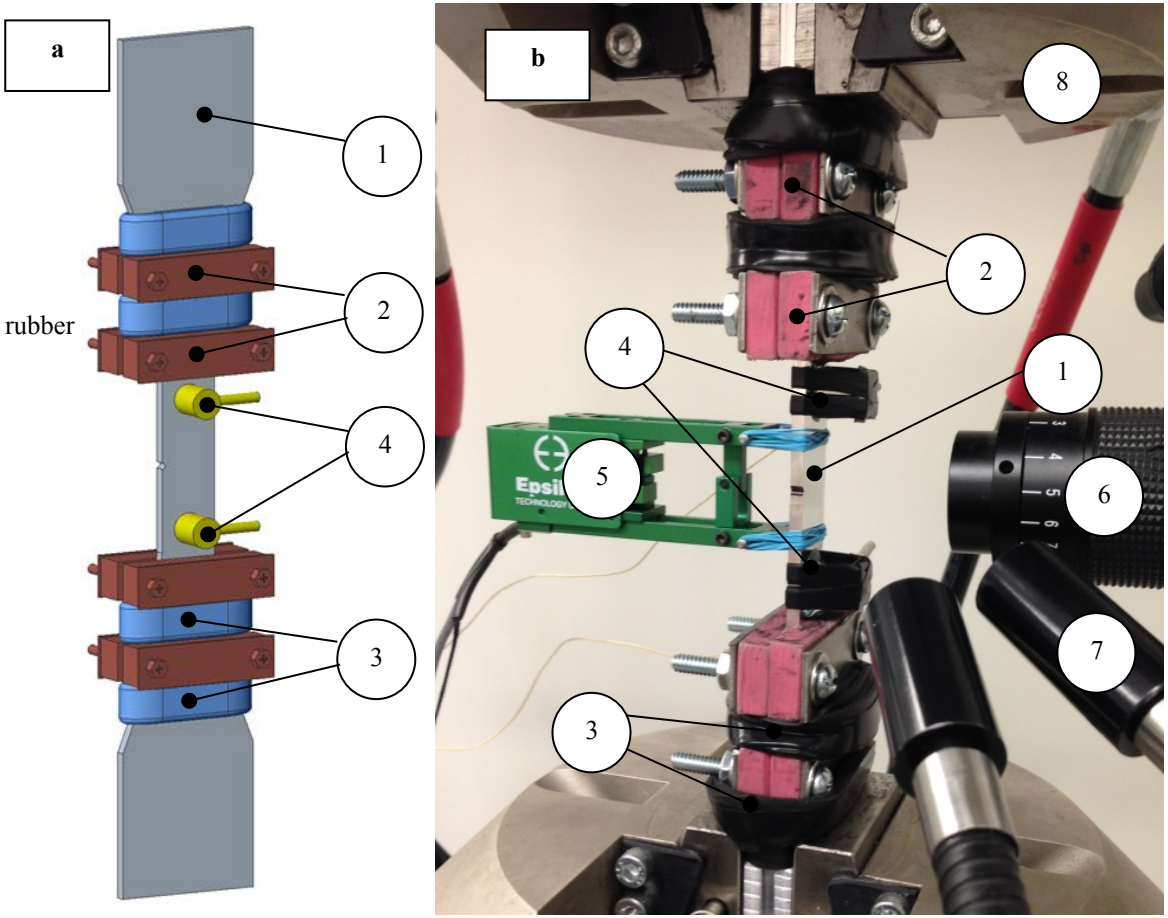






Fatigue Testing with AE (Cont.)

1. Specimen
2. Clamped styrene-butadiene rubber
3. Wrapped neoprene strip
4. Acoustic sensor
5. Extensometer
6. Optical microscope
7. External light source
8. Testing grip

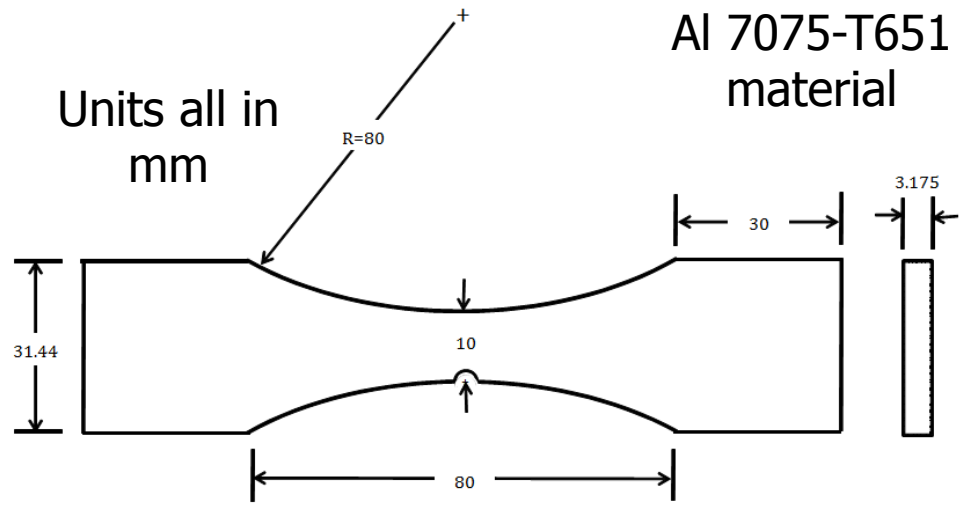


Fatigue Testing and Test Example

- Sample Information:
  - Test CSFs
    - Frequency = 2 Hz
    - Load Ratio = 0.5
    - Min/Max Load = 6500/13000 N
  - Material CSFs
    - Mean Grain Diameter – WBL(69.7,1.61)
    - Mean Inclusion Diameter – WBL(10.4,1.68)

Units all in mm

Al 7075-T651 material



Half circle notch measures 0.5 mm radius

- Data
  - 125 data collected (5 non-detections, 120 detections)
  - 108 training data, 12 validation data

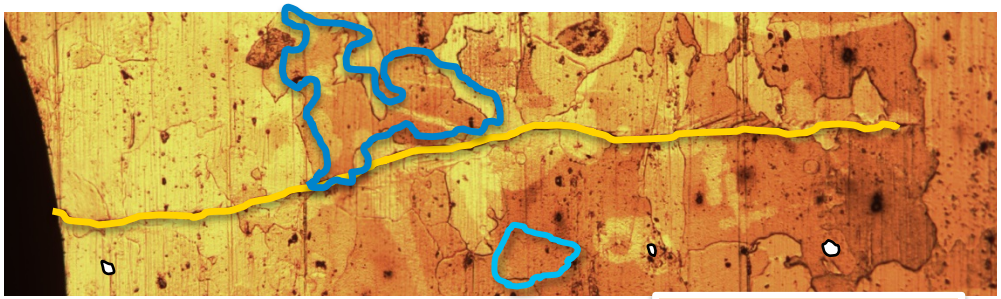
$$da/dN = \alpha\downarrow 1 \; dc/dN \uparrow \alpha\downarrow 2$$

Introduction of CSFs and Applications in Crack Growth Modeling

- **Crack Shaping Factors (CSFs)**:
  - Definition: Test or material based input properties that have direct bearing on the shape, length, and growth of the crack
  - Accounting minimize model uncertainties

Force,
Load Ratio,
and Load Frequency

Grain size



- Several growth and detection models were tested through probabilistic Bayesian parameter estimation procedures

Likelihood Function for Bayesian Analysis

- Find the model parameters with the following likelihood function,

$$l(D=0,1;a_i=1,\ldots,a_{n_D},x_i=1,\ldots,x_{n_D},x_j=1,\ldots,x_{m_{ND}}|A,B,P_{FD})=\prod_{i=1}^{n_D}[(1-P_{FD})POD(D=1|B,a_i>a_{lth})f(a_i|A,x_i)]\prod_{j=1}^{m_{ND}}[1-(1-P_{FD})\int_{a_{lth}}^{\infty}POD(D=1|B,a>a_{lth})f(a|A,x_j)da]$$

- Terms:
  - A – crack growth model parameters
  - B – crack detection model parameters
  - P_{FD} – probability of false detection
  - a – crack length at time point i or j
  - x – crack shaping factor vector pertaining to crack length a_i or a_j
  - n_D – number of detection data points $(x_i=1,\ldots,x_{n_D};a_i=1,\ldots,a_{n_D})$
  - m_{ND} – number of non-detection data points $(x_j=1,\ldots,x_{m_{ND}};a_j=1=0,\ldots,a_{m_{ND}}=0)$
  - POD – probability of detection CDF
  - f – crack propagation PDF
  - D – detection state ($D=1$ detected/$D=0$ not detected)

Crack POD Model Choices

- Lognormal POD Model
  - $POD(a|\zeta_0,\zeta_1,a_{lth})=\int_{a_{lth}}^{a} 1/(x-a_{lth})\sqrt{2\pi}\zeta_1^2 \exp\{-1/2\,[\ln(x-a_{lth})-\zeta_0/\zeta_1]^2\}dx$; $B=[\zeta_0,\zeta_1]$
- Log-logistic POD Model
  - $POD(a|\beta_0,\beta_1,a_{lth})=\exp[\beta_0+\beta_1\ln(a-a_{lth})]/1+\exp[\beta_0+\beta_1\ln(a-a_{lth})]$; $B=[\beta_0,\beta_1]$
- Logistic POD Model
  - $POD(a|\eta_0,\eta_1,a_{lth})=1-1+\exp(-\eta_0\,\eta_1)/1+\exp[\eta_0(a-\eta_1-a_{lth})]$; $B=[\eta_0,\eta_1]$
- Weibull POD Model
  - $POD(a|\alpha_0,\alpha_1,a_{lth})=1-\exp[-(a-a_{lth}/\alpha_0)^{\alpha_1}]$; $B=[\alpha_0,\alpha_1]$

Crack Growth Model Choices

- All crack growth models fit to the lognormal PDF distribution,
  - $f(a|A,x)=1/a\sigma\sqrt{2\pi}\ \exp[-1/2\ (\ln a-\ln g(A,x)/\sigma)^2]$

  where σ is the standard deviation

  - Log-Linear Growth Direct Model
    - $\ln a=b+m\ln N$
  - Acoustic Emission Growth Indirect Model
    - $a(N)=\{\blacksquare\alpha I(N)+\beta\&linear@\alpha I(N)^\beta\&power$

    where $I(N)$ is a function of AE amplitude and cumulative counts
  - Gaussian Process Regression (GPR) Growth Model
    - $a=g(x)=g([\blacksquare CSF_1\&CSF_2\&\cdots\&CSF_Q])$

More Details About the GPR Model

- $a \sim NOR[0, K([X], A)]$
  - where $K(.)$ is the $M \times M$ covariance matrix or kernel matrix that correlates a and $[X]$
  - Kernel matrix are made up of kernel functions $k(x_i, x_j, A)$ taking two sets of CSF data x_i and x_j and the Gaussian crack length model parameters A
  - $k(x_i, x_j, A) = A_1 + \sum_{q=1}^{Q} A_{q+1} x_{i,q} x_{j,q} + A_{2+2Q} \exp[-\sum_{q=1}^{Q} A_{q+Q+1} (x_{i,q} - x_{j,q})^2] + A_{4+2Q} \sin^{-1} A_{3+2Q} \sum_{q=1}^{Q} x_{i,q} x_{j,q} / \sqrt{(A_{3+2Q} \sum_{q=1}^{Q} x_{i,q} x_{j,q})^2 + A_{5+2Q}} \delta_{i,j}$ where $\delta_{i,j}$ is a Dirac function

Crack Growth Model Choices

- Input/output relation of GPR

Output:
Crack Length (μm)

Input:
Test and Material CSFs

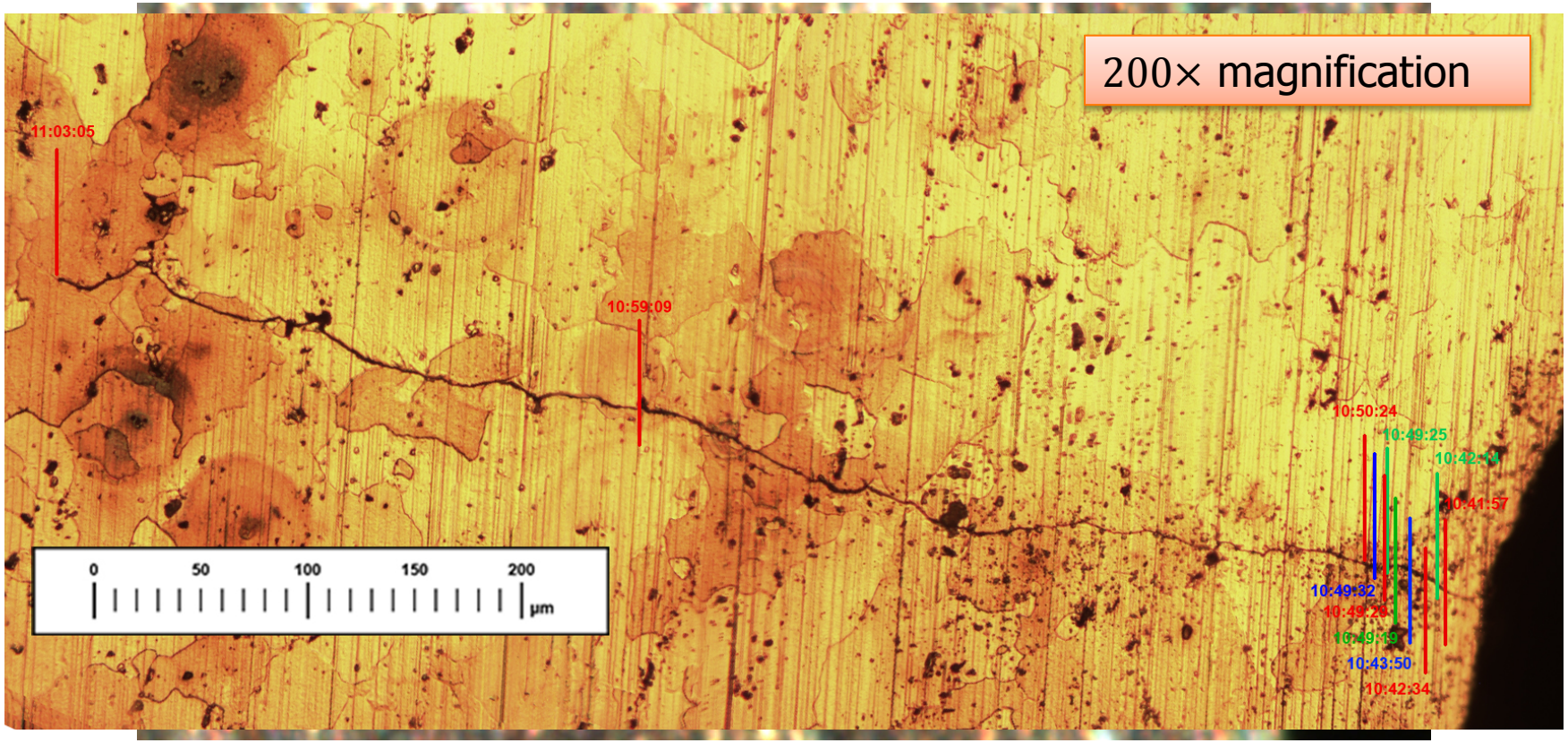
Model Parameters

$$\begin{bmatrix} \text{Crack Length (μm)} \\ 5.14 \\ 6.42 \\ \vdots \\ 441 \end{bmatrix} = g\left(\begin{bmatrix} \text{Growth Model Parameters} \\ a_1 \\ a_2 \\ \vdots \\ a_{108} \end{bmatrix}, \begin{bmatrix} \text{Fatigue Cycle} & \text{Min Load (N)} & \text{Max Load (N)} & \text{Load Ratio} & \text{Frequency (Hz)} & \text{Mean Grain Diameter α (μm)} & \text{Mean Grain Diameter β} & \text{Mean Inclusion Diameter α (μm)} & \text{Mean Inclusion Diameter β} \\ 2950 & 6500 & 13000 & 0.5 & 2 & 69.7 & 1.61 & 10.4 & 1.68 \\ 5984 & 6500 & 13000 & 0.5 & 2 & 69.7 & 1.61 & 10.4 & 1.68 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8324 & 6500 & 13000 & 0.5 & 2 & 69.7 & 1.61 & 10.4 & 1.68 \end{bmatrix} \right)$$

$$\vec{A}, \quad \begin{bmatrix} CSF_{1,1} & CSF_{1,2} & \cdots & CSF_{1,9} \\ CSF_{2,1} & CSF_{2,2} & \cdots & CSF_{2,9} \\ \vdots & \vdots & \vdots & \vdots \\ CSF_{108,1} & CSF_{108,2} & \cdots & CSF_{108,9} \end{bmatrix}$$

$$\vec{\delta F} = g\left(\vec{A}, \begin{bmatrix} CSF_{1,1} & CSF_{1,2} & \cdots & CSF_{1,9} \\ CSF_{2,1} & CSF_{2,2} & \cdots & CSF_{2,9} \\ \vdots & \vdots & \vdots & \vdots \\ CSF_{108,1} & CSF_{108,2} & \cdots & CSF_{108,9} \end{bmatrix}\right)$$

Fatigue Life Test Data Collection



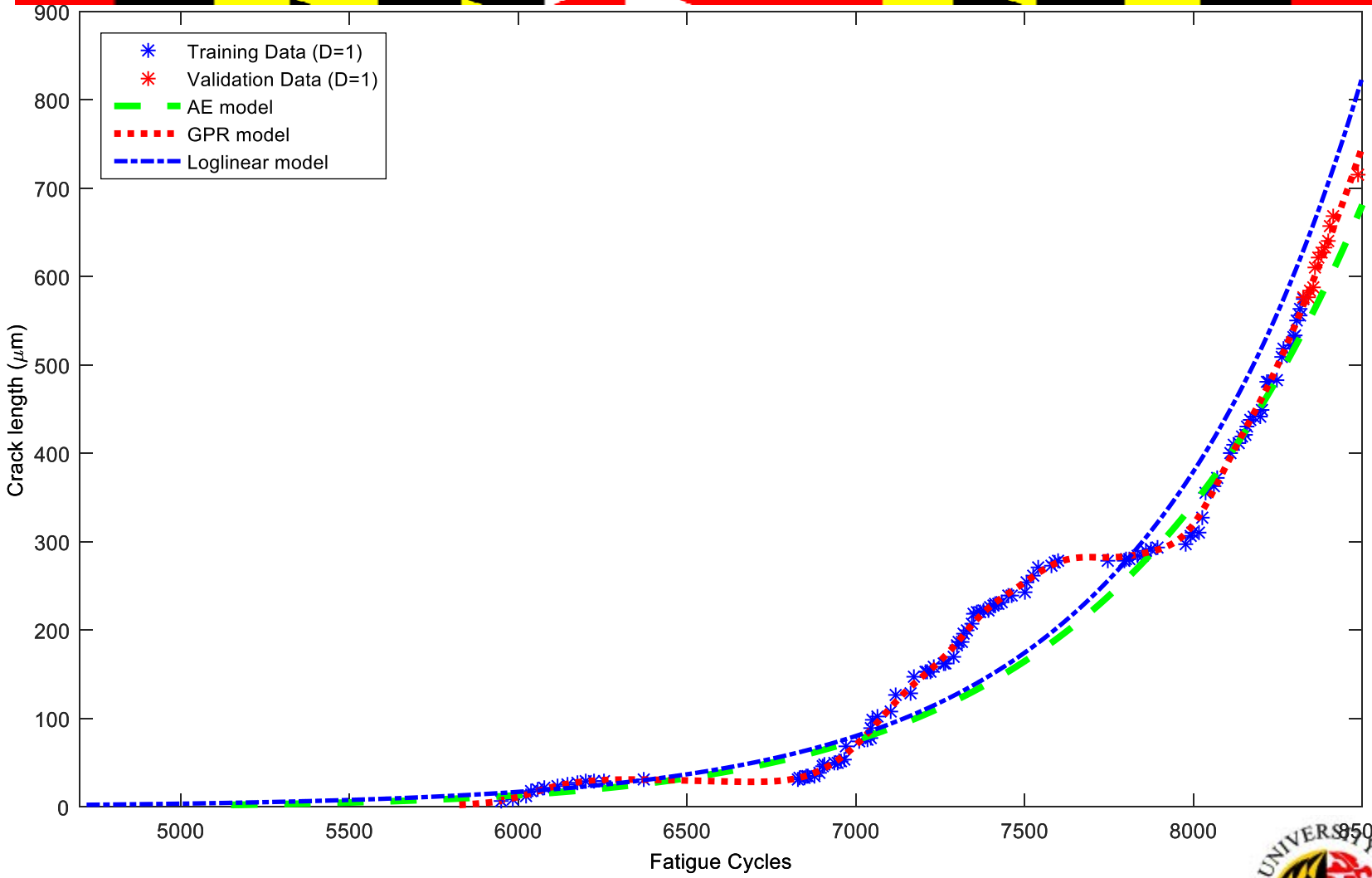
Measurement Error Considerations

	Log-Linear-Based Crack Length	AE-Based Crack Length
Mean Measurement Error	0.48	1.25
Model Error	51.9%	24.5%

- The model error between the true and estimated length is less for AE-Based crack length estimation than with the measured length estimation

- AE length measurement only overestimates the true length during early cycles

Results of Crack Growth Models Considered

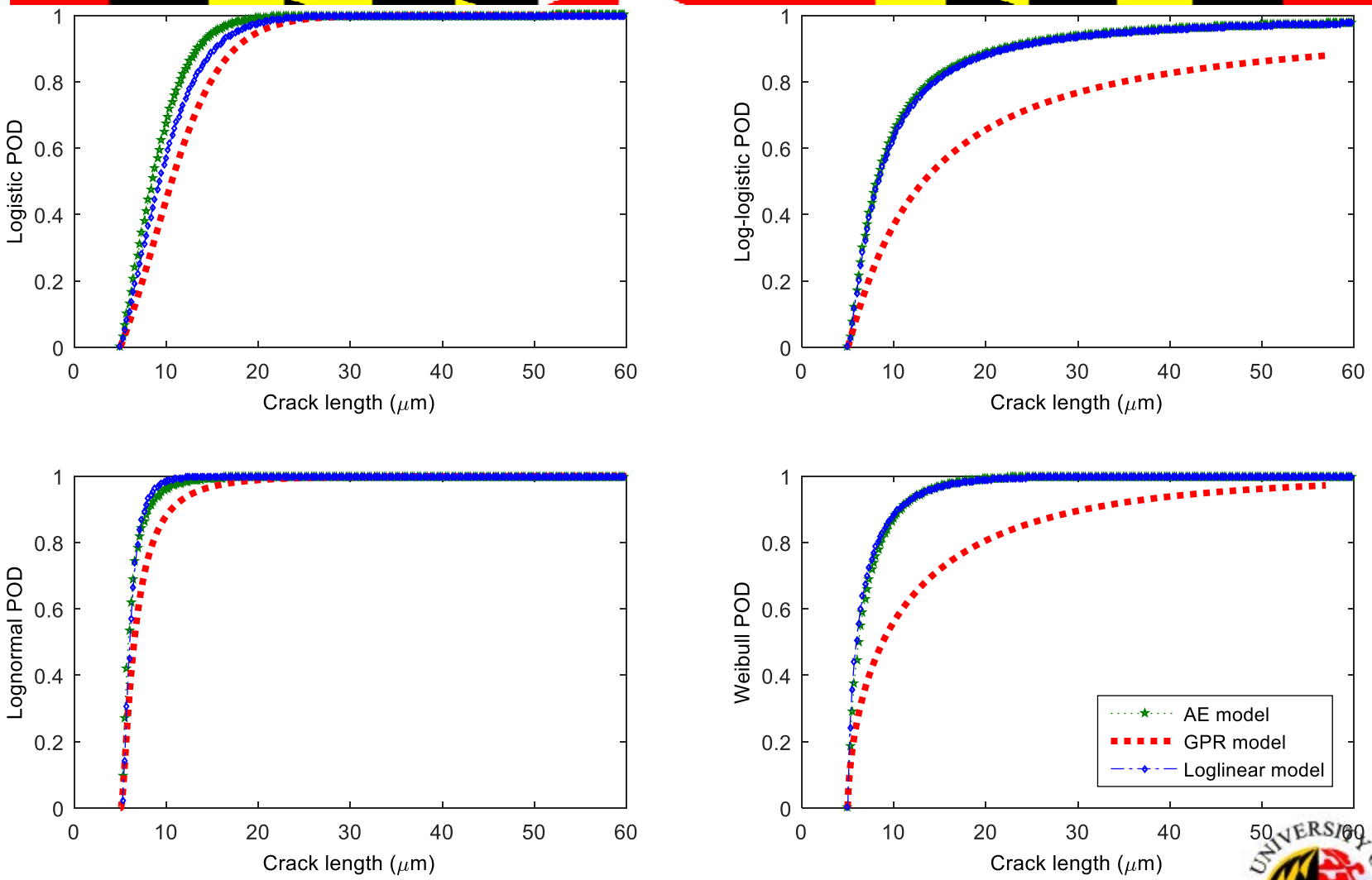


Crack Growth/Detection Model Results

Growth Model\POD		Lognormal	Logistic	Log-Logistic	Weibull		AVERAGE
NMSE	Log-Linear	2.4	2.3	2.3	1.9		2.2
	AE	0.37	0.30	0.29	0.28		0.31
	GPR	0.042	0.041	0.041	0.041		0.042

- The GPR/Log-logistic pair has the smallest normalized mean square error (NMSE) of validation data
- Number of CSFs used understandably correlates to lower NMSE
  - GPR – 9 CSFs
  - AE – 3 CSFs
  - Log-linear – 1 CSF

POD Model Results



Conclusions

- AE-Based model
  - Has a lower error value than that of measured length model error
  - Has improved over previous studies from 42% to 24.5%
- By implementing the GPR on 12 CSFs pairs the model error could be reduced by a large sum
- The number of CSFs used directly impact the fitness and realism of the crack length model.

THANK YOU!

