Probabilistic Physics-of-Failure: An Entropic Perspective

Presentation
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The Center for Risk and Reliability

Outline of this Talk

- A word about me and This Talk
- Reliability Engineering Timeline
- Frontiers in Reliability Engineering Research
- Probabilistic Physics of Failure
- Entropy as the Science of Reliability
- Conclusions

A Word About My Research and This Talk

ABOUT MY RESEACH

- My research is in Probabilistic Risk Assessment, Probabilistic Physics-of-Failure, Probabilistic Fracture Mechanics, Deep Learning-Based Prognosis and Health Management, Uncertainty Management

ABOUT THIS TALK

- This talk has been given in different forms in the past
- This video version is prepared for the benefit your esteemed conference and for all reliability researchers that follow my work worldwide. It is made available from my Website http://modarres.umd.edu.
- I have been keenly interested to understand and describe reliability engineering within a fundamental physical sciences
- This would not necessarily a departure from the traditional views that explain reliability solely based on evidences in form of raw data and information from reliability tests or field observations. Rather, it is an adjunct to the traditional methods to elucidate and enrich the reliability analyses and results.

Timeline of Reliability Engineering

- Initiatives in 1950's
  - Weakest link
  - Exponential life model
  - Reliability Block Diagrams (RBDs)
- Exponential Distribution Retreat in 1960's
  - Birth of Physics of Failure (POF)
  - Uses of other distributions
  - Reliability growth
  - Life testing
  - Failure Mode and Effect Analysis (FMEA)
- Logic Models: Fault Tree Analysis in 1970'S
  - Probabilistic Risk Assessment (PRA)
  - Common Cause Failures (CCFs)
  - Uncertainty analysis

Timeline (Cont.)

- Accelerated Life and Degradation Testing in1980's
- Rebirth of Physics-of-Failure in 1990's
  - Probabilistic Physics-of-Failure (PPOF)
  - Time varying accelerated tests (e.g., Step-Stress Test)
  - Highly Accelerated Life Testing (HALT)
- Hybrid Reliability and Prognosis Models in 2000's
  - Combined logic models, POF models and probabilistic models (e.g. BBN)
  - Prognosis and Health Management (PHM) methods
  - Powerful simulation tools (MCMC, Recursive Bayes and Particle Filtering)
- Exploring Fundamental Sciences of Reliability in 2010 and Beyond 2020
  - Thermodynamics and Entropy
  - Data science and predictive Analytics (treating big data, deep learning methods in reliability)
  - Autonomous systems and robots
  - Infrastructure and cyber-physical systems

Frontier Research Areas in Reliability Engineering

- Probabilistic Physics-of-Failure (PPoF)
  - More than 50-years of history in PoF (More Recently PPoF)
  - Empirical model for Unit-Specific reliability assessment
  - Simulation-based reliability
- Hybrid System Reliability
  - Combined Techniques: NN, CNN, BBN, DBN, DFT, DET, Markov and Semi-Markov, FEM and FDM, FM, RBD, etc.
- Sensor-Based / Big Data Reliability Analysis
  - Data Fusion, Predictive Analytics, Deep Learning, Natural Language, Detection Probability, Measurement Models
- Fundamental Sciences of Reliability Engineering
  - 2nd Law of thermodynamics and entropy
  - Statistical mechanics and Gibbs entropy
  - Information entropy and Kullback–Leibler Divergence (KLD)

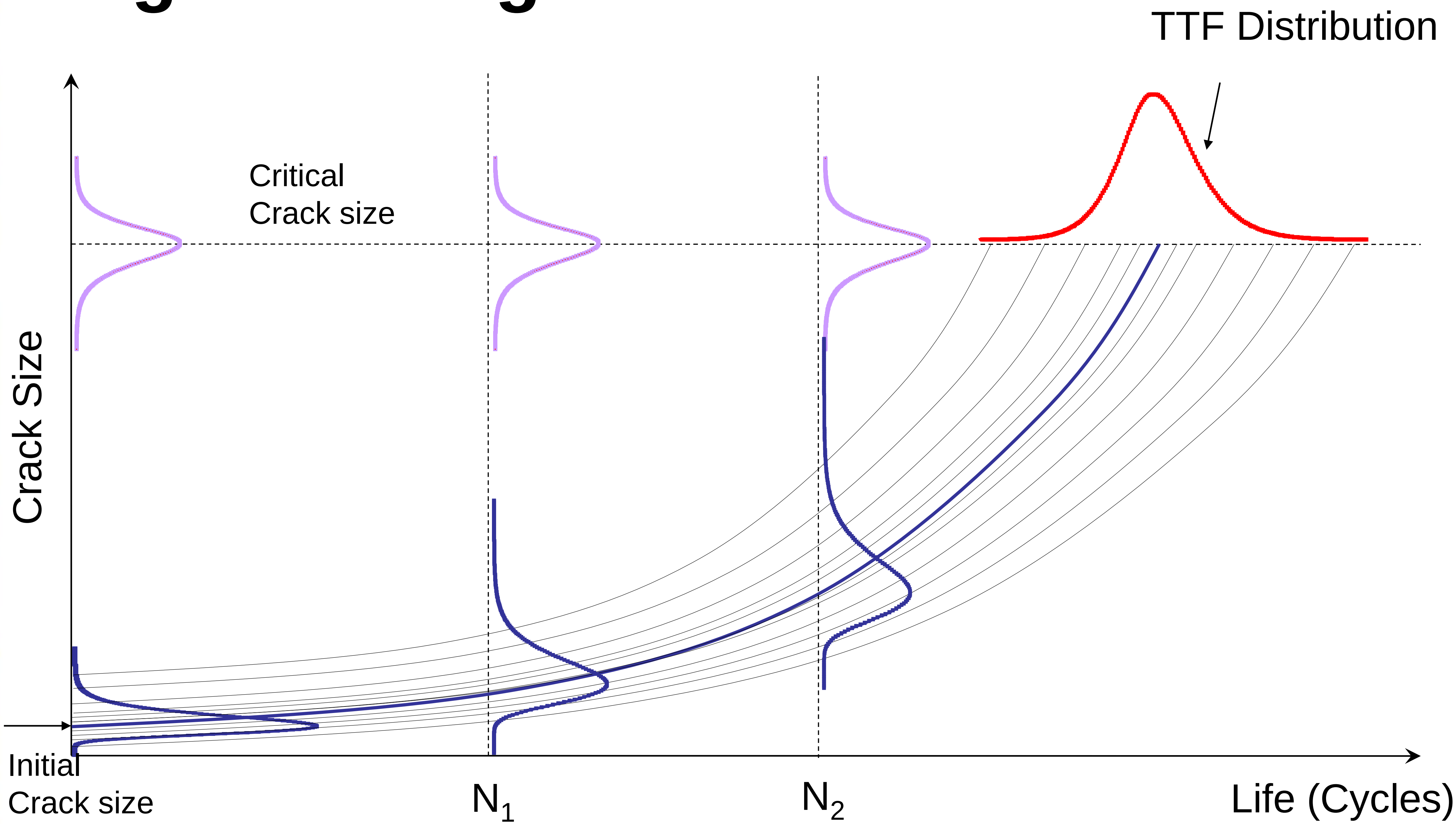
Physics of Failure (POF) and Probabilistic POF (PPOF)

POF is an engineering approach to reliability assessment that uses simulation of the physical models of failures developed based on the empirical science of failure mechanisms such as fatigue, fracture, wear, and corrosion.

$$\boldsymbol{time - \mathrm{to} - failure}$$

- PPOF is unit-specific models, but expensive to build.

An Example of a PPOF Fatigue Damage-Endurance Model



Objectives of Entropic Damage in Reliability

- Describe damage resulted from failure mechanisms within an entropic framework
- Understand sources of irreversible energy dissipation measurements in the fatigue process, i.e. mechanical, thermal, and acoustic
- Develop entropy for each dissipation measurement representing damage or current state of material, based on thermodynamic, information, and statistical mechanics theorems.
- Search for applications to Reliability Engineering: Prognosis and Health Management (PHM) of structures

Motivation

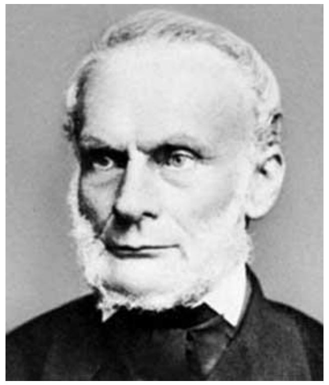
Common definitions of damage are based on <u>observable markers</u> of damage which vary at different geometries and scales

- Macroscopic Markers of Damage (e.g. changes in elastic modulus, pit densities, weight loss)
- Macroscopic Fatigues Markers include: crack length, reduction of modulus, reduction of load carrying capacity
- Issue: When markers of damage observed 80%-90% of life has been expended

An Entropic Theory of Damage: A Fundamental Science of Reliability

- Failure mechanisms leading to degradation share a common feature at a deeper level: ***Dissipation of Energy***
- Dissipation (or equivalently entropy generation)≅Damage

Degradation mechanisms → Damage → Dissipation energies → Entropy generation

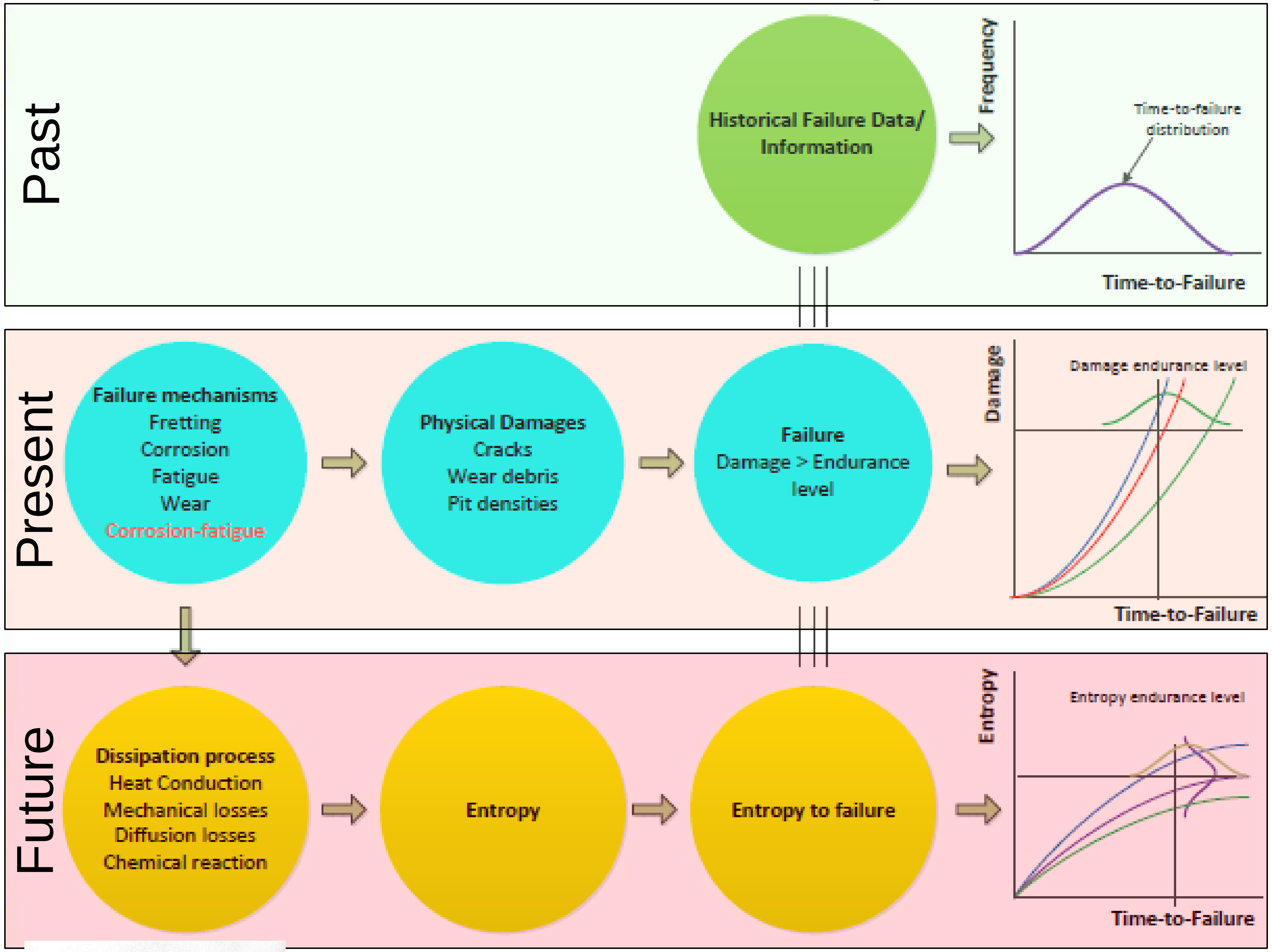


Rudolf Clausius
1822 –1888

Failure[1] occurs when the accumulated total entropy generated exceeds the entropic-endurance of the unit

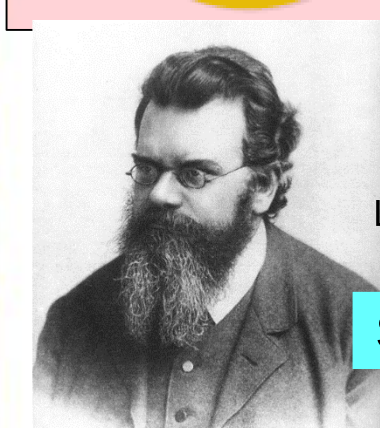
- Entropic-endurance describes the capacity of the unit to withstand entropy
- Entropic-endurance of identical units is equal
- Entropic-endurance of different units is different
- Entropic-endurance to failure can be measured (experimentally) and involves stochastic variability

Thermodynamics as a Science of Reliability



Why Entropy?

- ✓ Entropy can model multiple competing degradation processes leading to damage
- ✓ Entropy is independent of the path to failure ending at similar total entropy at failure
- ✓ Entropy accounts for complex synergistic effects of interacting degradation processes
- ✓ Entropy is scale independent



Ludwig Boltzmann
1844-1906

Statistical Mechanics Entropy

Thermodynamics as a Science of Reliability (Cont.)

Second Law of Thermodynamics: In an isolated system, entropy will always increase until it reaches a maximum value.

Second Law of Thermodynamics (Statistical Mechanics Version): In an isolated system, the system will always progress to a macrostate that corresponds to the maximum number of microstates.

All damages resulting from failure mechanisms share a common feature: Dissipation of Energy.

Dissipation: a fundamental determinant of irreversibility can be described well within the context of non-equilibrium thermodynamics.

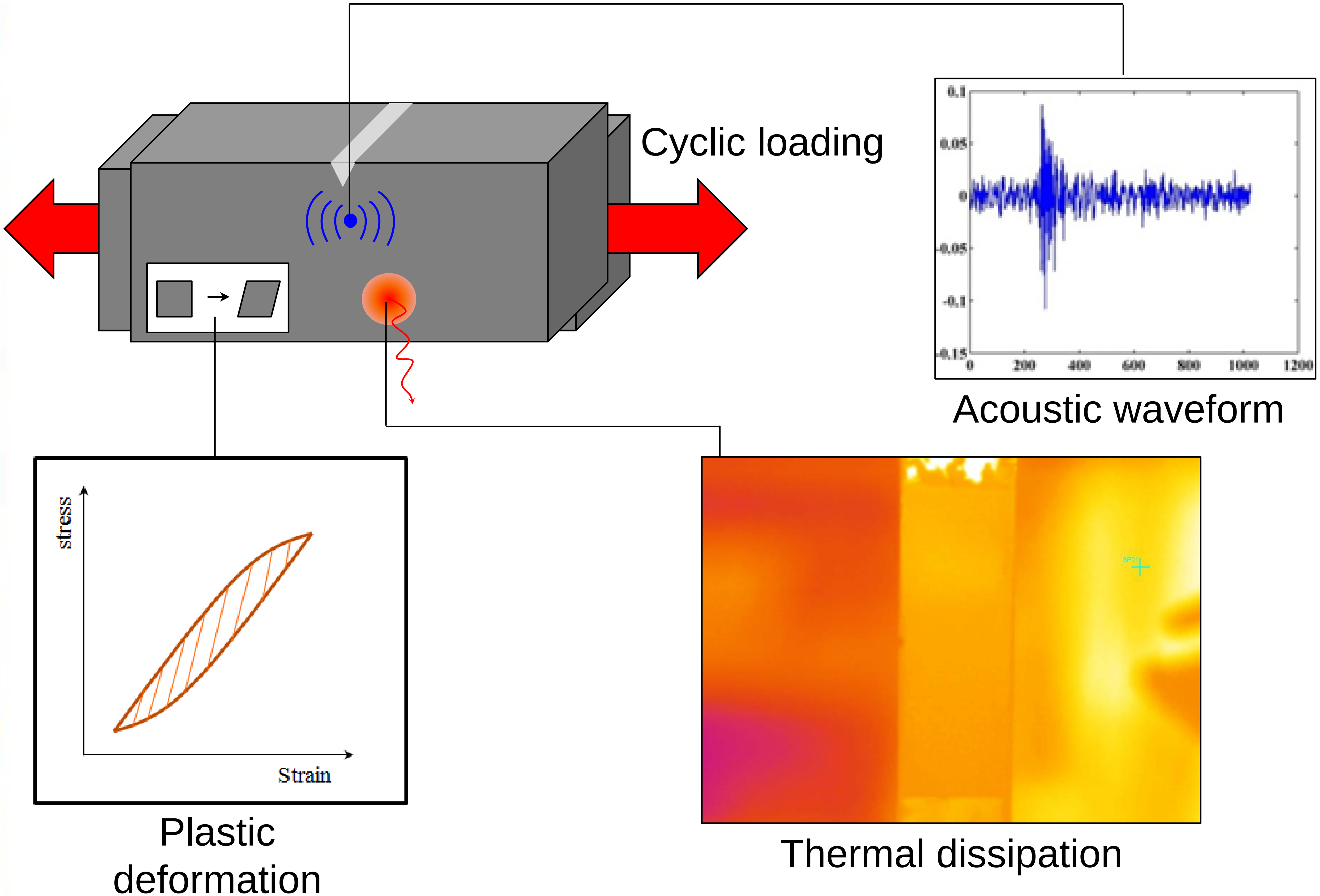
We will show that the "Maximum Entropy Value" is the point of materials failure (In the context of reliability we name it <u>Entropic Endurance to Failure</u>.

Entropic Approaches to Represent Damage

Dissipation (Measurement) Source
Plastic Deformation
Thermal
Acoustic Emission

Entropic Approach	Related Equation
Second Law of Thermodynamics	$\sigma = \frac{1}{T^2}\boldsymbol{J}_q.\nabla T - \Sigma_{k=1}^{n}\boldsymbol{J}_k\left(\nabla\frac{\mu_k}{T}\right) + \frac{1}{T}\boldsymbol{\tau}:\dot{\boldsymbol{\epsilon}}_p + \frac{1}{T}\Sigma_{j=1}^{r}v_j\,A_j + \frac{1}{T}\Sigma_{m=1}^{h}c_m\boldsymbol{J}_m(-\nabla\psi)$
Information Theory (Shannon)	$S = -\sum p_i \log p_i$
Statistical Mechanics (Crooks)	$D(P_F\|\|P_R) = \sum P_{F,i}\ln\frac{P_{F,i}}{P_{R,i}}$

Sources of Dissipation in Fatigue Process



Ali Kahirdeh and M.M. Khonsari, Energy dissipation in the course of the fatigue degradation: Mathematical derivation and experimental quantification, International Journal of Solids and Structures 77 (2015): 74-85

Thermodynamics Entropy

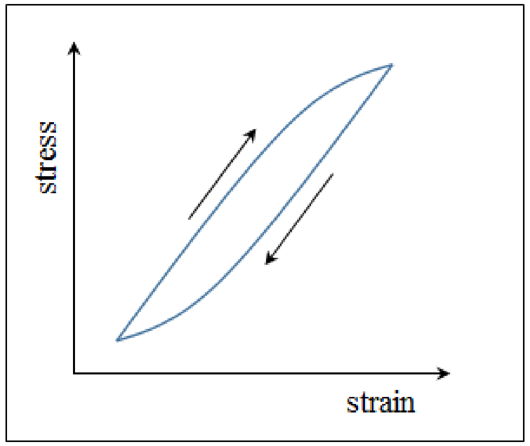
- Entropy generation σ involves a thermodynamic force, X_i, and an entropy flux, J_i as:

$$\sigma = \Sigma_{i,j} X_i J_i(X_j)\ ; \quad (i, j=1,\ldots, n)$$

- Entropy generation of important dissipation phenomena leading to damage:

$$\sigma = \overbrace{\frac{1}{T^2}\boldsymbol{J}_q.\nabla T}^{\text{Thermal}} + \overbrace{\Sigma_{k=1}^{n}\boldsymbol{J}_k\left(\nabla\frac{\mu_k}{T}\right)}^{\text{Diffusion}} + \overbrace{\frac{1}{T}\boldsymbol{\tau}:\dot{\boldsymbol{\epsilon}}_p}^{\text{Plastic deformation}} + \underbrace{\frac{1}{T}\Sigma_{j=1}^{r}v_j\ A_j}_{\text{Chemical reaction}} + \underbrace{\frac{1}{T}\Sigma_{m=1}^{h}c_m\boldsymbol{J}_m(-\nabla\psi)}_{\text{External fields}}$$

$\boldsymbol{J}_n$ $(n = q, k, and\ m)$ = thermodynamic fluxes due to heat conduction, diffusion and external fields, T=temperature, μ_k = chemical potential, v_i =chemical reaction rate, $\boldsymbol{\tau}$ =stress tensor, $\dot{\boldsymbol{\epsilon}}_p$ =the plastic strain rate, A_j =the chemical affinity or chemical reaction potential difference, ψ =potential of the external field, and c_m =coupling constant *, **



$$\Delta S_{total} = \frac{W^{diss}}{T} = \frac{Hysteresis\ Area}{T}$$

Hysteresis Area: From stress-strain analysis
T: From surface temperature measured by infrared camera or thermocouple

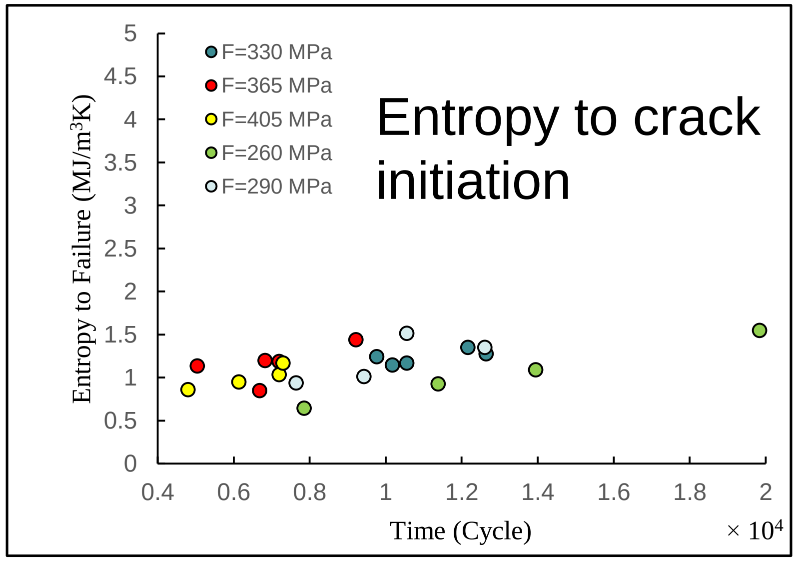
Thermodynamics Entropy in Fatigue Damage (Cont.)

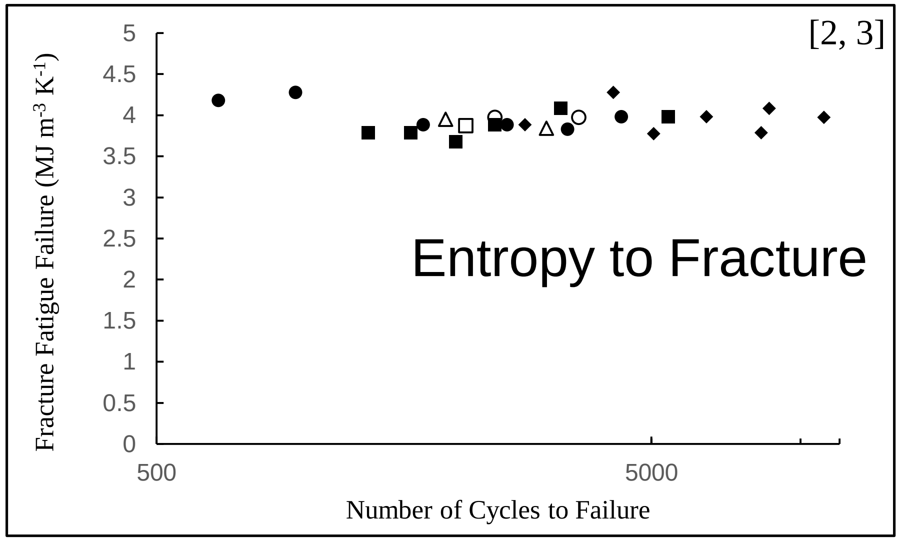
$$\sigma = \frac{1}{T^2} J_q \cdot \nabla T + \frac{1}{T} \tau : \dot{\epsilon}_p$$

$$\sigma = \sum_{i=1}^{m} X_i J_i$$

Product of thermodynamic forces and fluxes

[1]

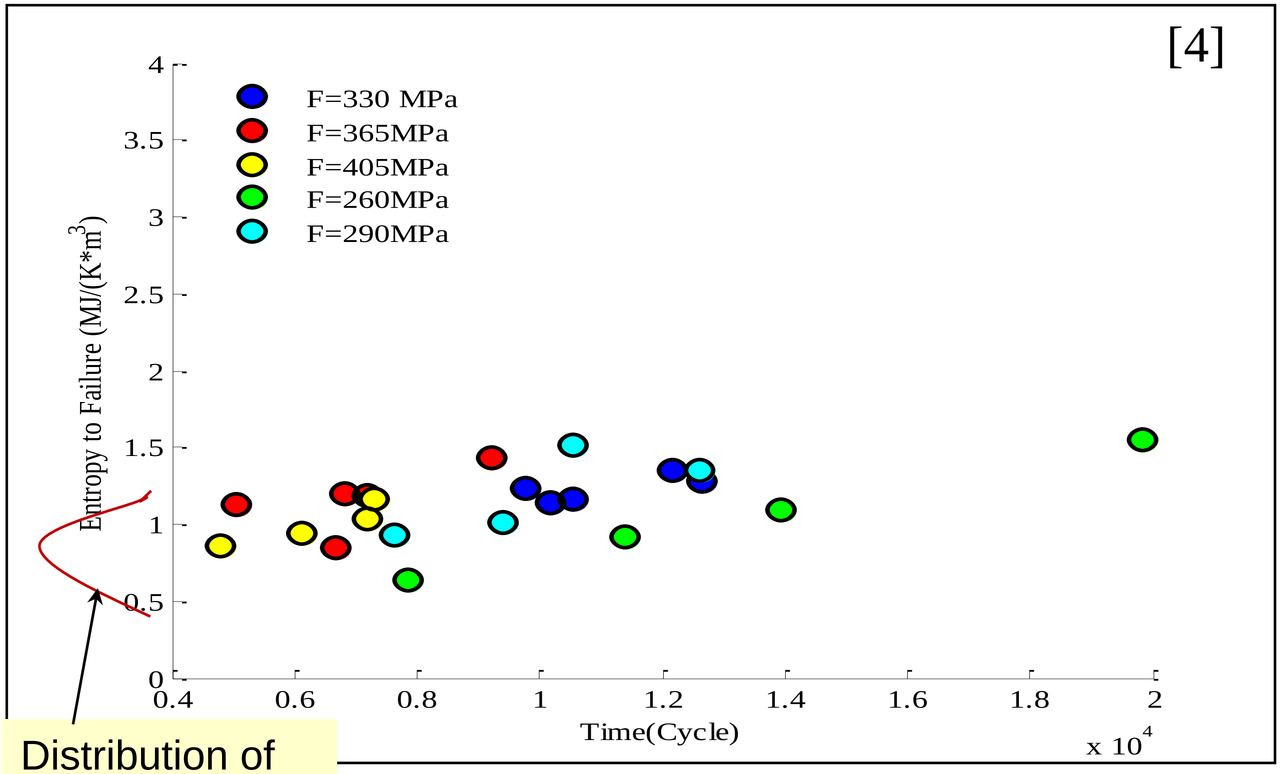




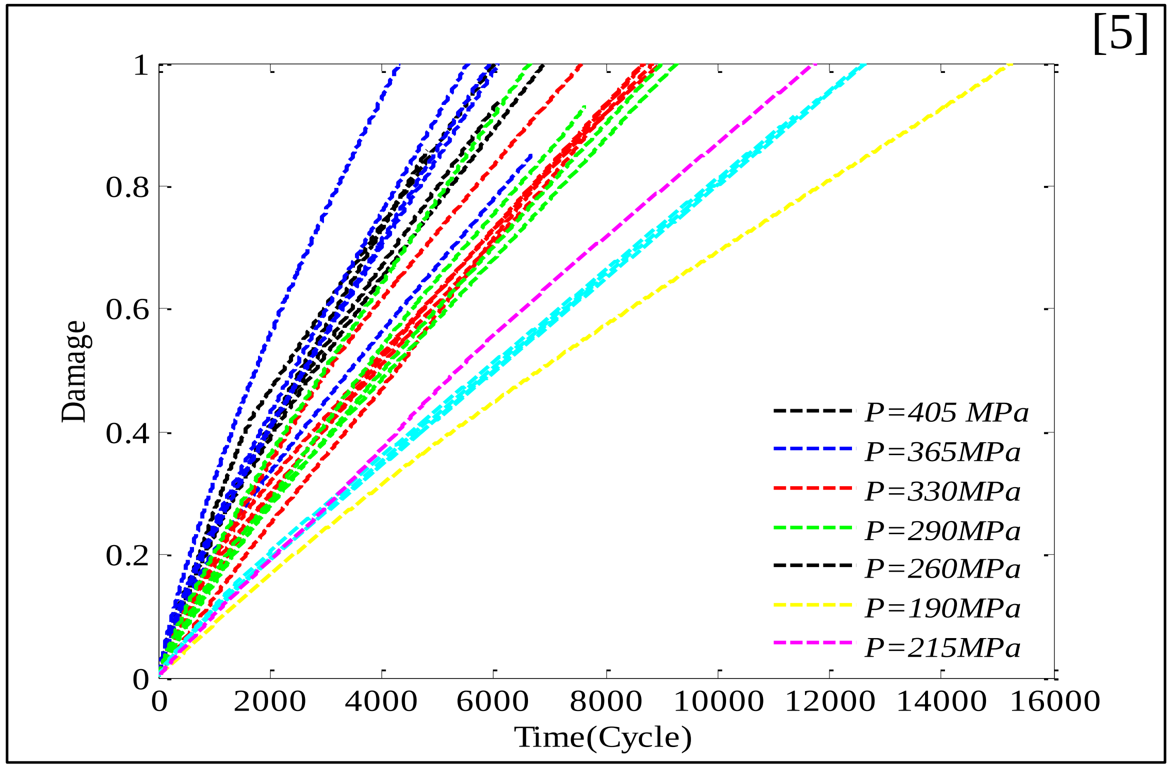
[1] Anahita Imanian and Mohammad Modarres, A Thermodynamic Entropy Approach to Reliability Assessment with Application to Corrosion Fatigue, Entropy 17.10 (2015): 6995-7020
[2] M. Naderi et al., On the Thermodynamic Entropy of Fatigue Fracture, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 466.2114 (2009): 1-16
[3] M. Naderi et al., Thermodynamic Analysis of Fatigue Failure in a Composite Laminate, Mechanics of Material 46 (2012): 113-122

Thermodynamics Entropy in Fatigue Damage (Cont.)

- Similarity of the total entropy-to-failure for all tests supports the entropic theory of damage offered proposed
- More tests needed to reduce the epistemic uncertainties and future confirm the theory

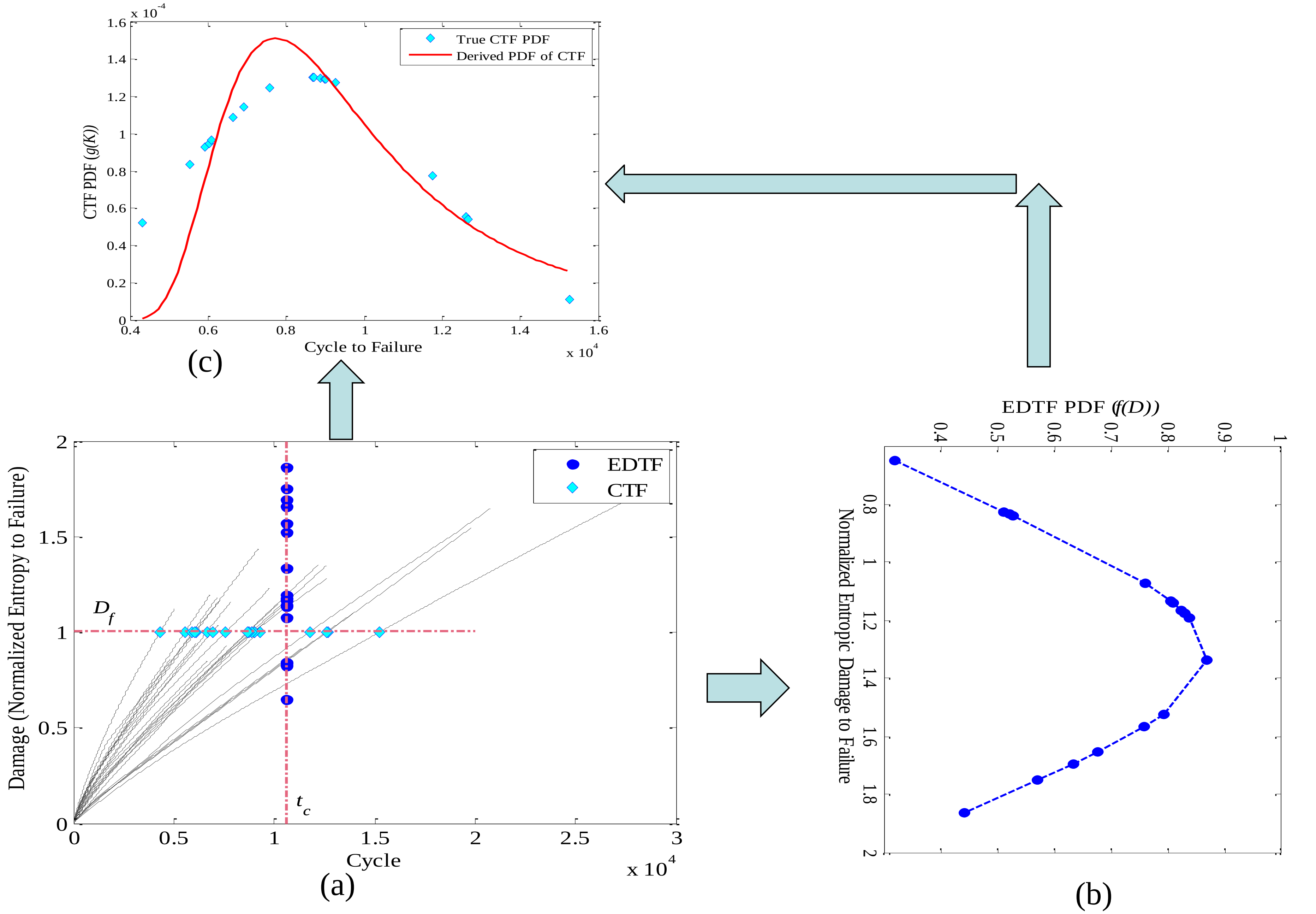


Distribution of entropic-endurance



[4] Mohammad Modarres, A General Entropic Framework of Damage: Theory and Applications to Corrosion-Fatigue, Structural Mechanics TIM 2015, 25-26 June 2015, Falls Church, VA, USA
[5] Anahita Imanian and Mohammad Modarres, Structural Health Monitoring, 2018, Vol. 17(2) 240-254

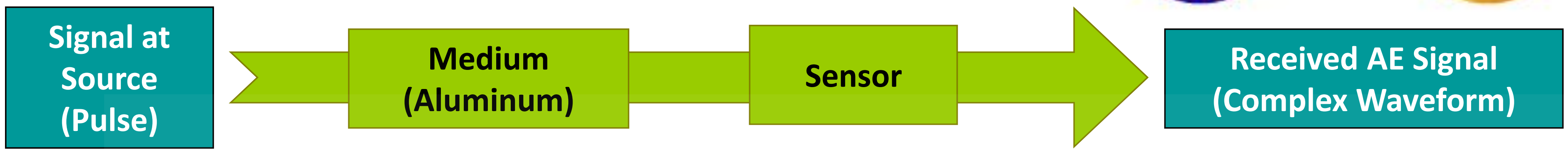
Entropic-Based Reliability

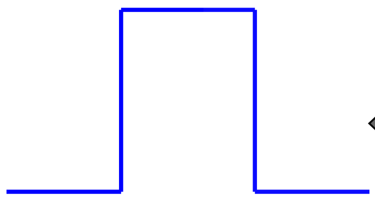


Thermodynamics as a Fundamental Science of Reliability, A. Imanian, M. Modarres, Int. J. of Risk and Reliability, Vol.230(6), pp.598-608. DOI: 10.1177/1748006X16679578.(2016).

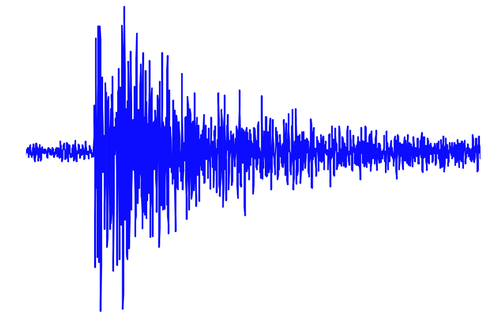
Anahita Imanian and Mohammad Modarres, A Thermodynamic Entropy Approach to Reliability Assessment with Application to Corrosion Fatigue, Entropy 17.10 (2015): 6995-7020

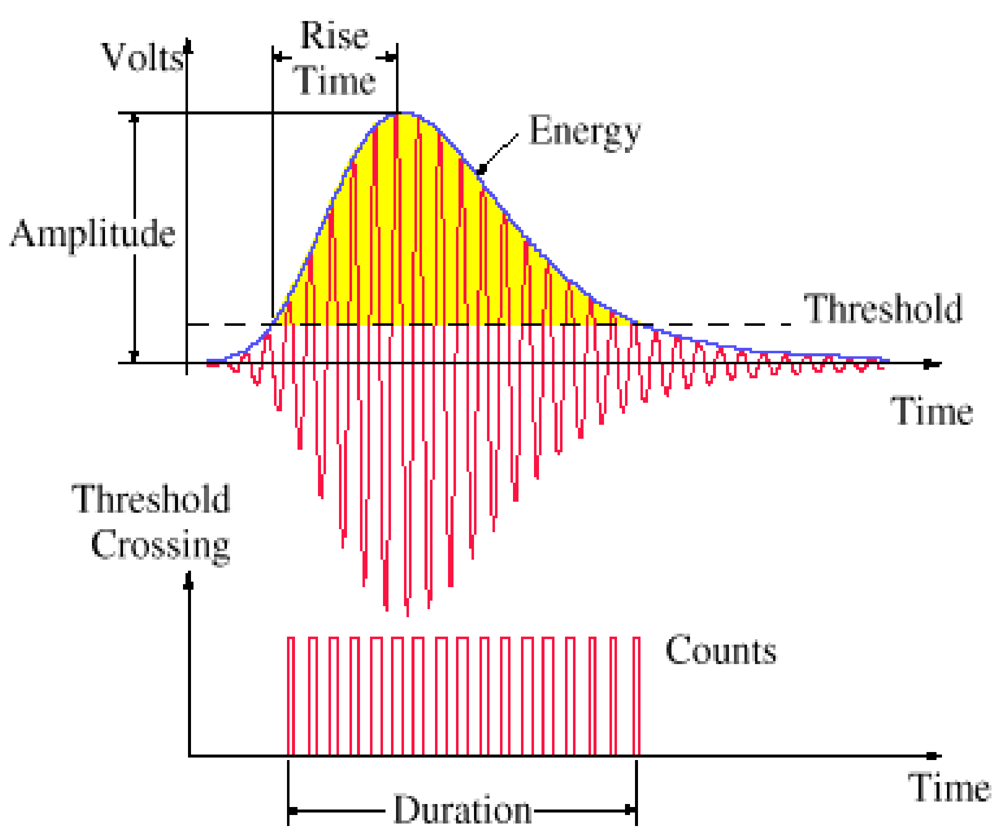
Example of Information Entropy (Acoustic Emission)





Deconvolution of the measured voltage signal from the sensor to evaluate the properties of the source event is extremely difficult.





AE Features

- Amplitude
- Energy
- Rise time
- Counts (Threshold crossing)
- Frequency content
- Waveform shape

Example of Information Entropy (Acoustic Emission)

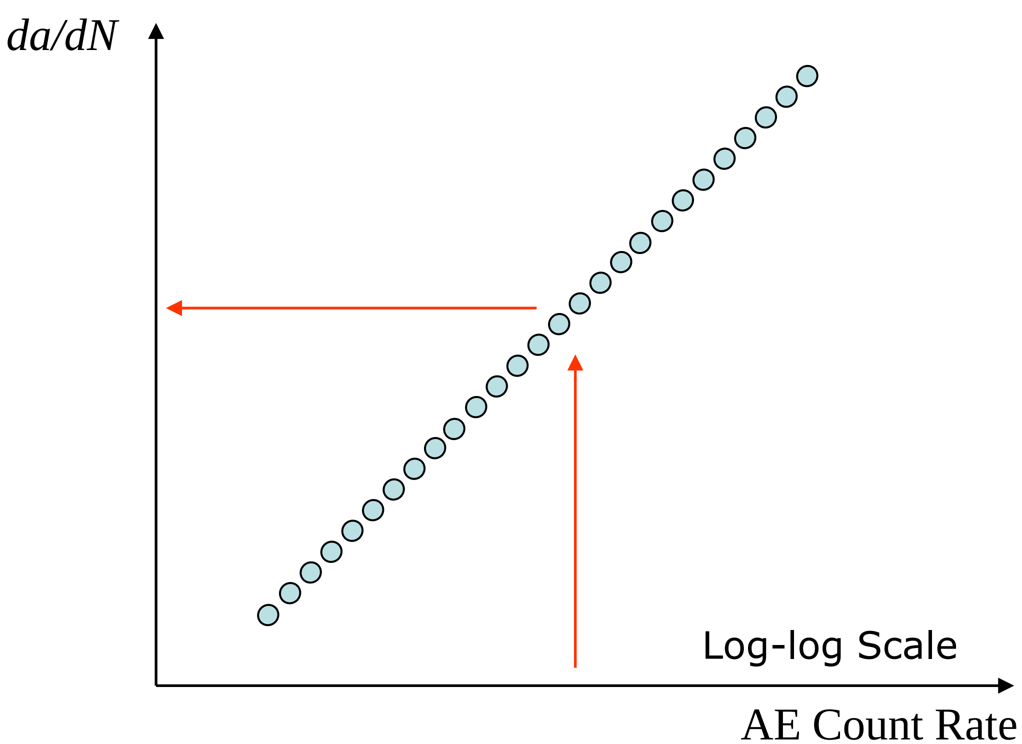
Acoustic Emission

- Amplitude
- Energy
- Rise time
- Counts
- Frequency Waveform

correlation →

Fatigue

- Crack Growth Rate
- $\Delta K = f(\text{stress, crack size})$

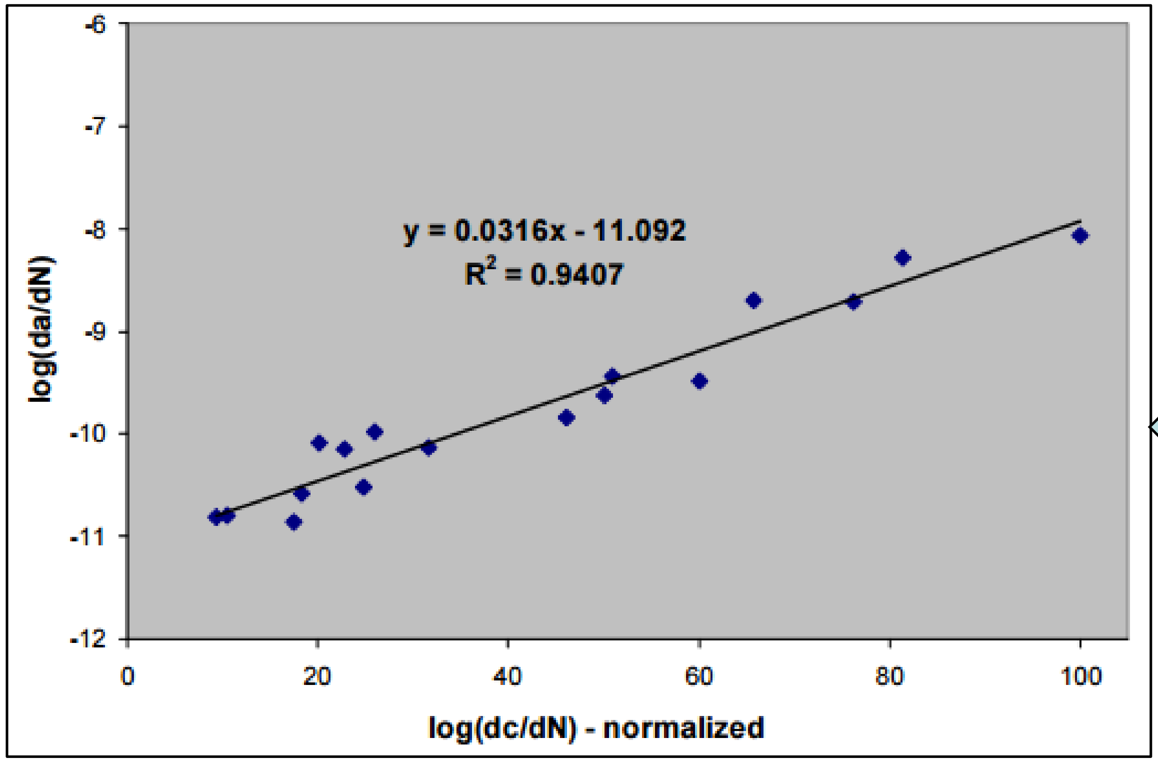


$$\log\left(\frac{da}{dN}\right) = \beta_1 \log\left(\frac{dc}{dN}\right) + \beta_2$$

One can estimate *da/dN*, given β_1, β_2 and AE count rate

[1] Bassim, M.N., St Lawrence, S. & Liu, C.D., 1994. Detection of the onset of fatigue crack growth in rail steels using acoustic emission. *ENG FRACT MECH*, 47(2), 207-214.

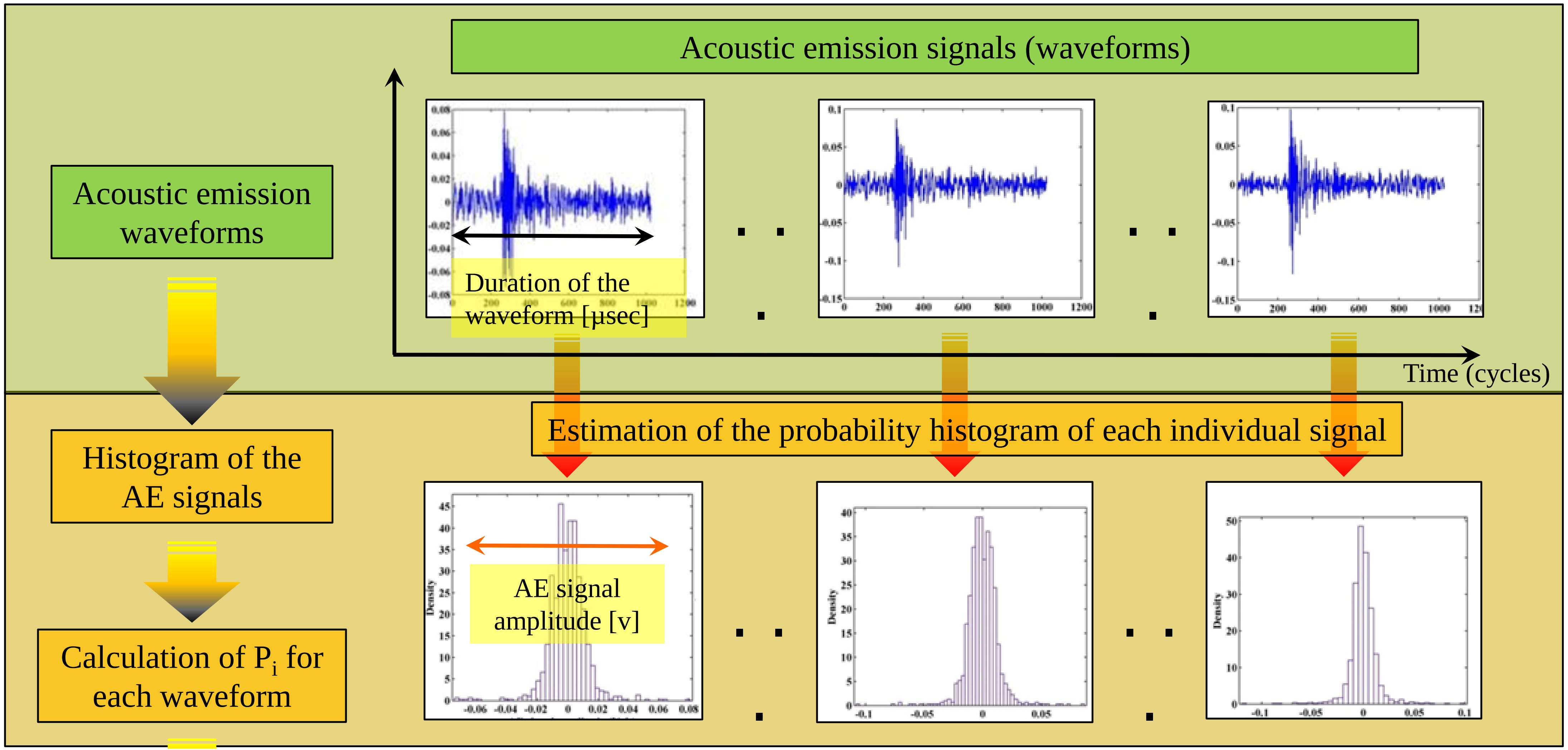
Example of Information Entropy (Acoustic Emission)



Crack growth correlation with an AE feature: AE count

A. Keshtgar and M. Modarres, Acoustic Emission-Based Fatigue Crack Growth Prediction, Reliability and Maintainability Symposium (RAMS), 2013 Proceedings-Annual, p.1-5

Entropy of AE Information



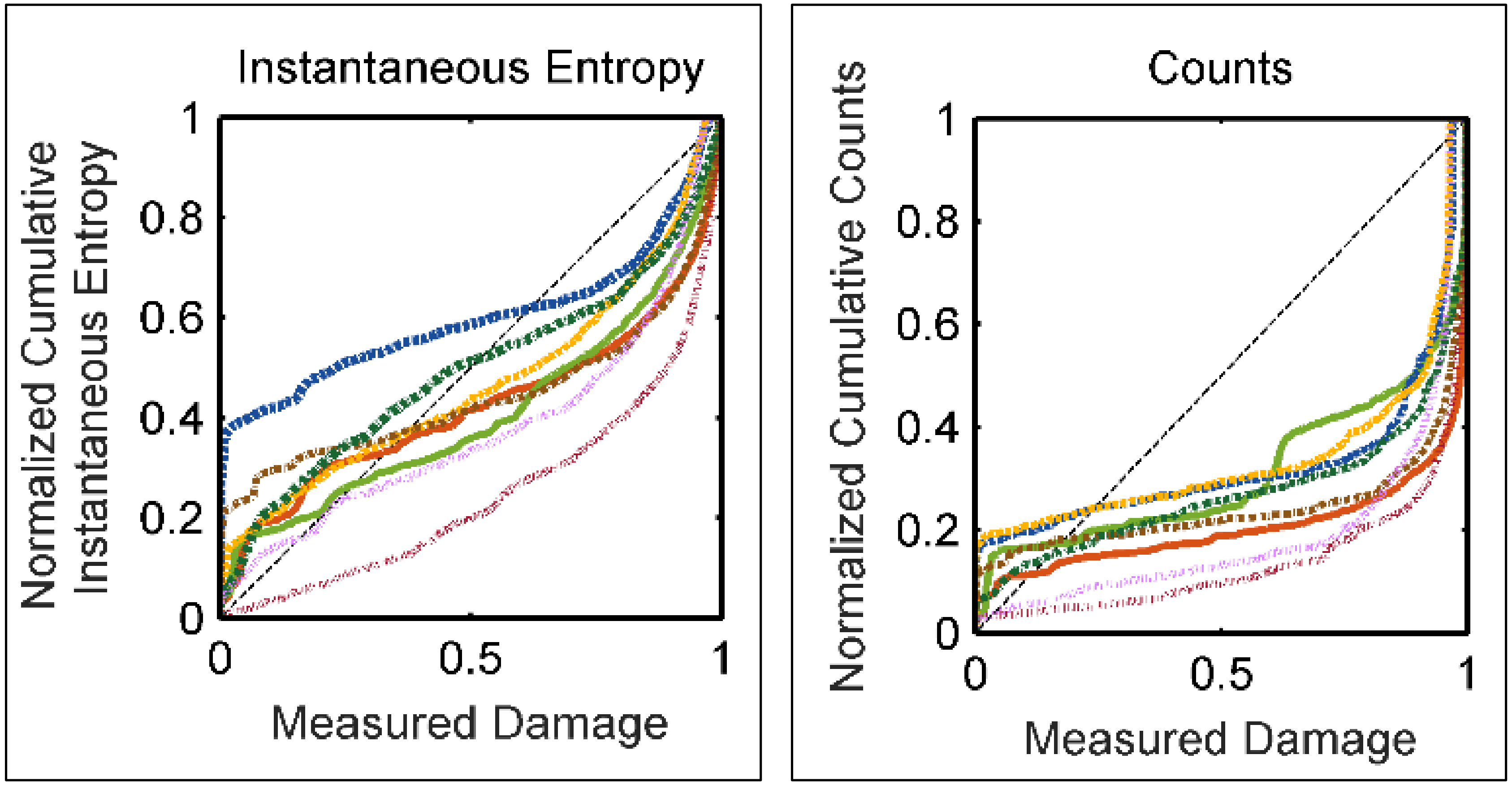
Probability distribution (P_i) → Entropy

$$S_I = \sum_{i=1}^{n} p_i \, log_2\left(\frac{1}{p_i}\right) = -\sum_{i=1}^{n} p_i \, log_2(p_i)$$

$$S_R = \sum_{i=1}^{n} p_i \, log \, \frac{p_i}{p_{R,i}}$$

Entropy of AE Information

- Cumulative AE information entropy better correlates with the measured damage in terms of changes in the elastic modulus



[6]

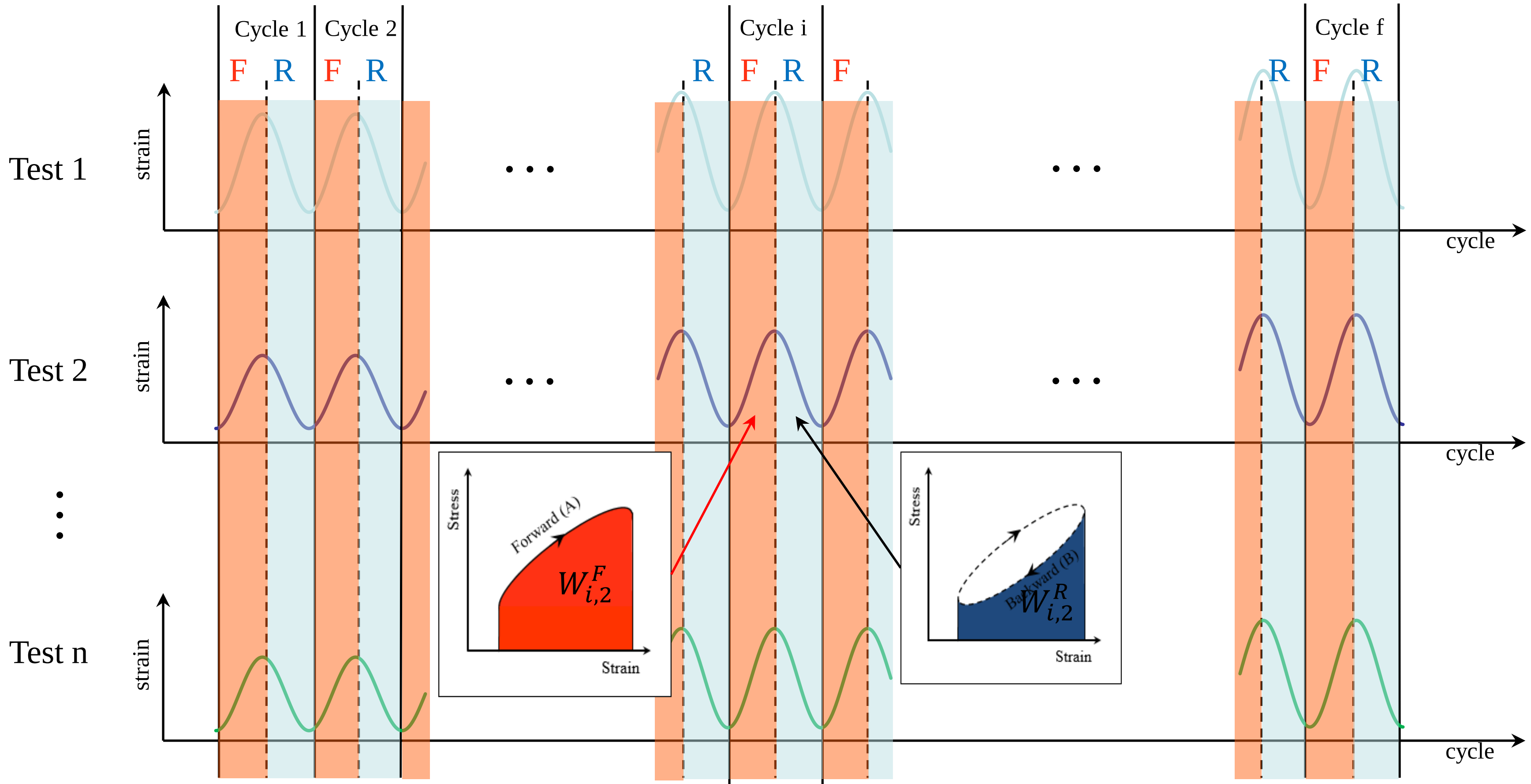
[6] Sauerbrunn, C. M., et al. "Damage Assessment Using Information Entropy of Individual Acoustic Emission Waveforms during Cyclic Fatigue Loading." *Applied Sciences* 7.6 (2017): 562

Entropy in Statistical Mechanics

- Relative entropy (Kullback-Leibler Divergence)

$$D(P_F||P_R) = \sum P_{F,i} \ln \frac{P_{F,i}}{P_{R,i}} = \Delta S_F^{Total} \quad [7]$$

- KLD equals the total entropy in a forward process or a reverse process.
- KLD is computed by repeating many similarly conditioned fatigue tests to measure forward / reverse work distributions

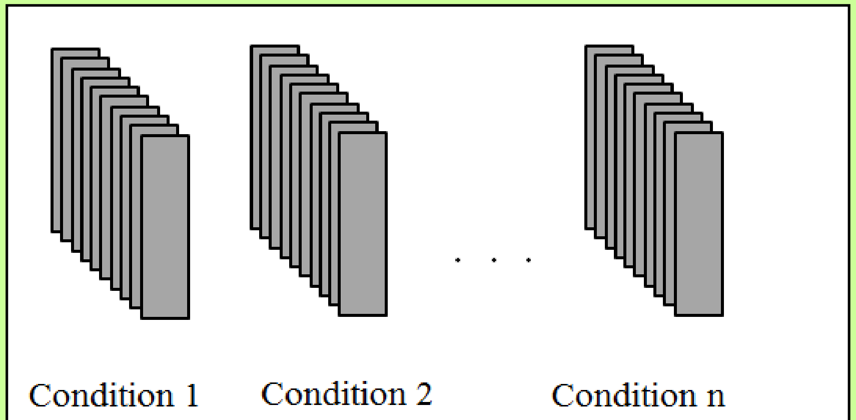


[7] Gavin E Crooks and David A Sivak, Measures of trajectory ensemble disparity in nonequilibrium statistical dynamics, Journal of Statistical Mechanics: Theory and Experiment, doi: 10.1088/1742-5468/2011/06/P06003

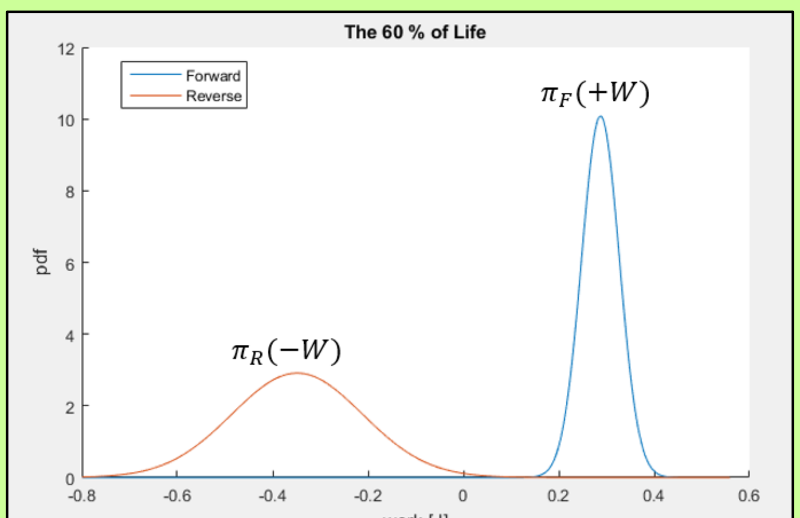
Entropy in Statistical Mechanics (Cont.)

- Analysis Procedure

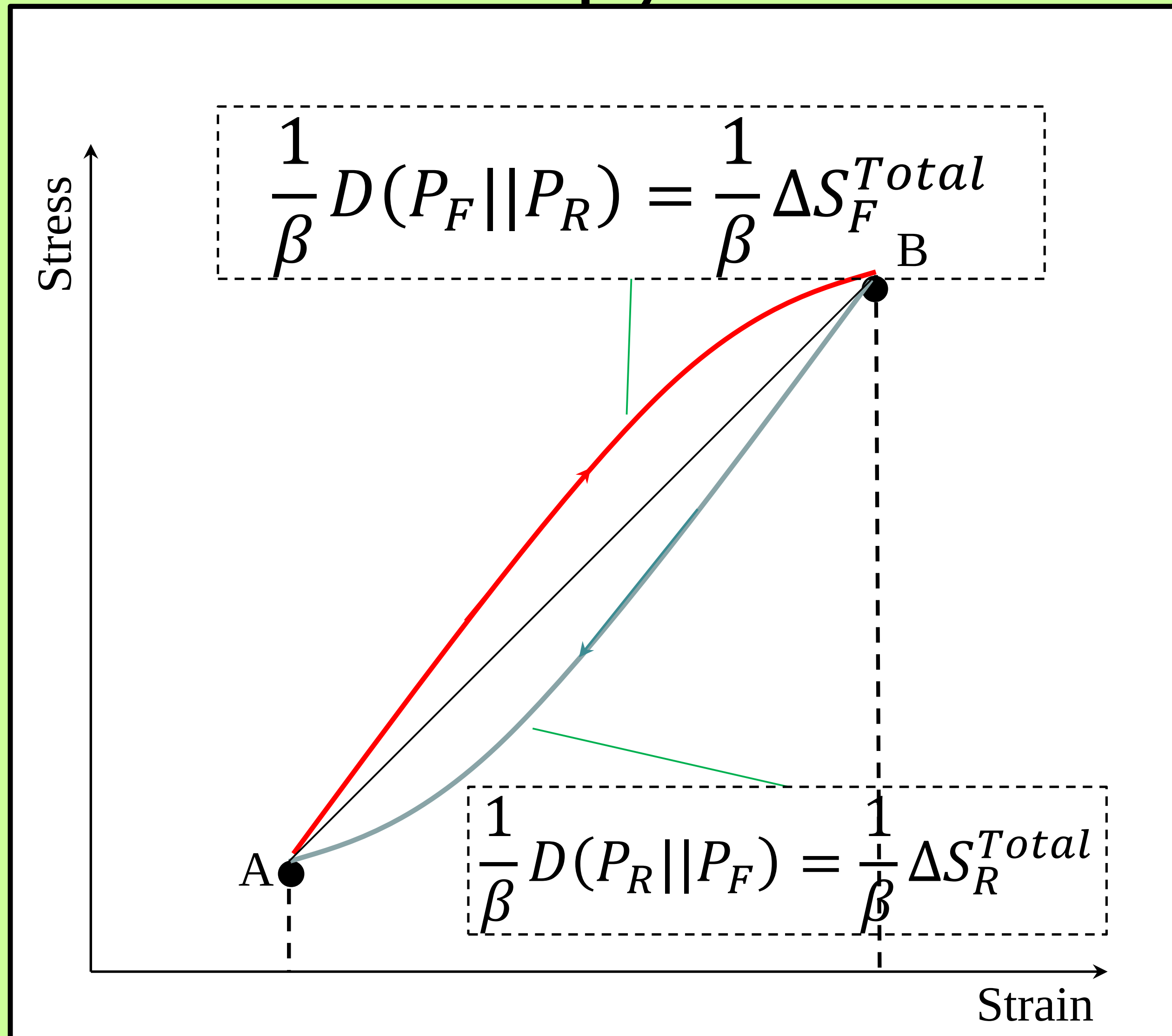
Multiple tests



Distributions of Forward and Backward



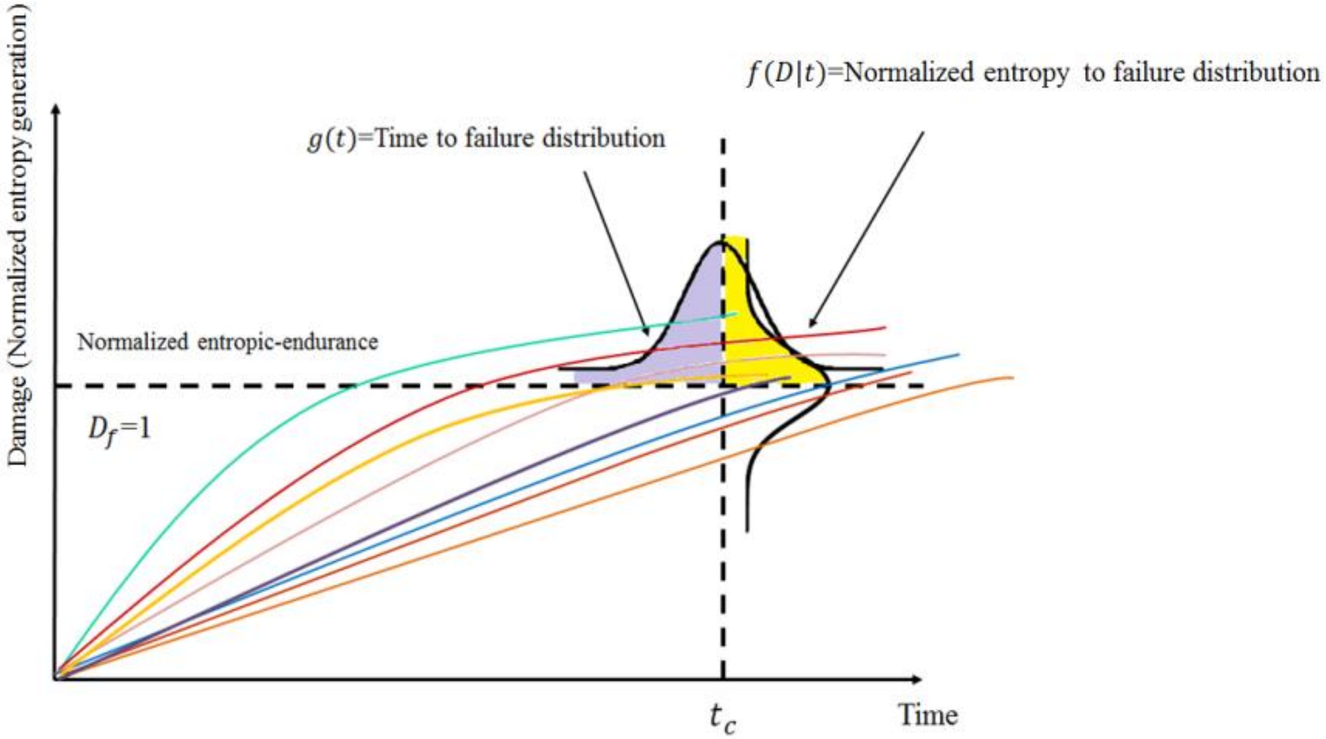
Relative entropy



An Entropic PPOF Perspective

- Materials, environmental, operational and other types of variabilities in degradation forces impose uncertainties on the total entropic damage

- Assuming a constant *entropic-endurance*, D_f



- The reliability function can be expressed as [9]

$$P_r(T \leq t_c) = \int_0^{t_c} g(t)dt = 1\text{-} \int_0^{D_f=1} f(D)dD$$

$$R(t_c) = 1 - P_r(T \leq t_c) = \int_0^{D_f=1} f(D)dD$$

T_c=*Current operating time;* $g(t)$=*distribution of time-to-failure,* $f(D|t)$= *distribution of damage at t*

[8] Thermodynamics as a Fundamental Science of Reliability, A. Imanian, M. Modarres, Int. J. of Risk and Reliability, Vol.230(6), pp.598-608. DOI: 10.1177/1748006X16679578.(2016).

Conclusions

- Overview of reliability history, POF and PPOF is provided
- Three different entropic damage measures were investigated based on: classical thermodynamics, statistical mechanics and information theory
- A thermodynamic theory of damage proposed and tested
- Damage model derived from the 2nd law of thermodynamics was used to develop models of reliability of materials
- The theory verified through multiple fatigue and corrosion-fatigue tests
- The proposed theory offered a more fundamental non-empirical PPOF model of damage and allowed incorporation of all interacting dissipative processes
- Statistical mechanics-based entropic damage theory is a promising approach
- Information entropy is useful in PHM when coupled with data analytics algorithms

For more details visit my website for more detail and publicly available information
http://modarres.umd.edu

Thank you for your attention!