# Examples of Uncertainty Analysis in Risk-Informed Applications Involving Physical Processes, Material Degradation and Fracture

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# Summary

- Types of Uncertainty Assessments
- Example 1: Probabilistic Model Development From Data Considering Limited Information and Data: Piping subject to Pitting Corrosion & Corrosion-Fatigue
- Example 2: Model Uncertainty Assessment: Complex Fire Model Codes.
- Example 3: Probabilistic Fatigue Failure Model Development with Diverse Type of Data
- Example 4: Assessment of Airframe Integrity Involving Fatigue Crack Model Assessment Based on Inspection and NDI Observations
- Example 5: A Bayesian Approach to Strain Life Model Development



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# Uncertainty is Certain!

## Example 1

Probabilistic Model Development From Data Considering Limited Information and Data: Piping subject to Pitting Corrosion & Corrosion-Fatigue



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# Problem Statement

Pitting and Fatigue leading to Pipe Failure - Ductile Iron

Corrosion Inside the Pipe

**Event**

Degradation of pipeline due to corrosion and fatigue

↓

Leaks & ruptures

↓

Unwanted plant shutdowns

**Consequences**

- Cost due to inspection & repair
- Cost due to loss of production
- Cost due to impact on environment

**Problem:**
Cost of corrective maintenance

**Solution:**
Predictive maintenance

Pictures from : (http://your-tech-assistant.ettex-pipe.com)



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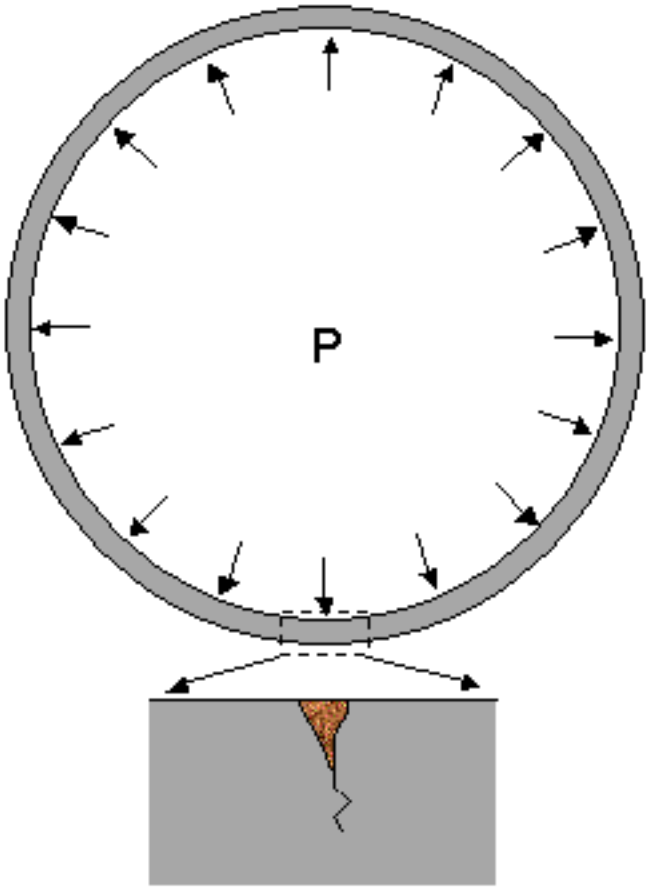
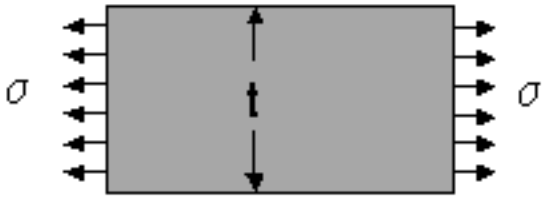
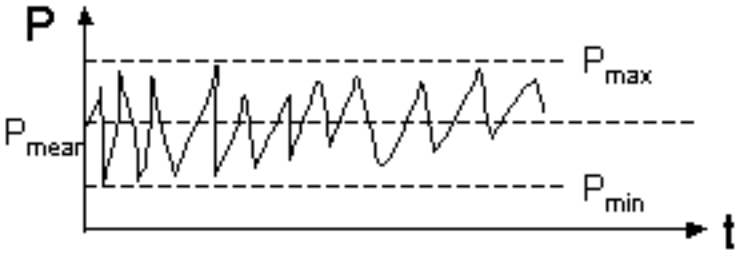
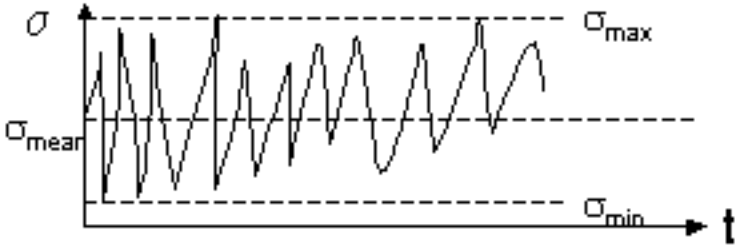
# Corrosion-Fatigue in Pipeline




$$\sigma = \frac{PD}{2t}; \frac{t}{D} << 1; D = diameter; t = tickness$$



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# Modeling Approach

Pit Nucleation → Pit Growth → Crack Nucleation (Pit to crack transition) → Crack Growth

**Corrosion is Dominant** | **Fatigue is Dominant**

**The criterion for transition :**

$$\left(\frac{da}{dt}\right)_{crack} \geq \left(\frac{da}{dt}\right)_{pit}$$



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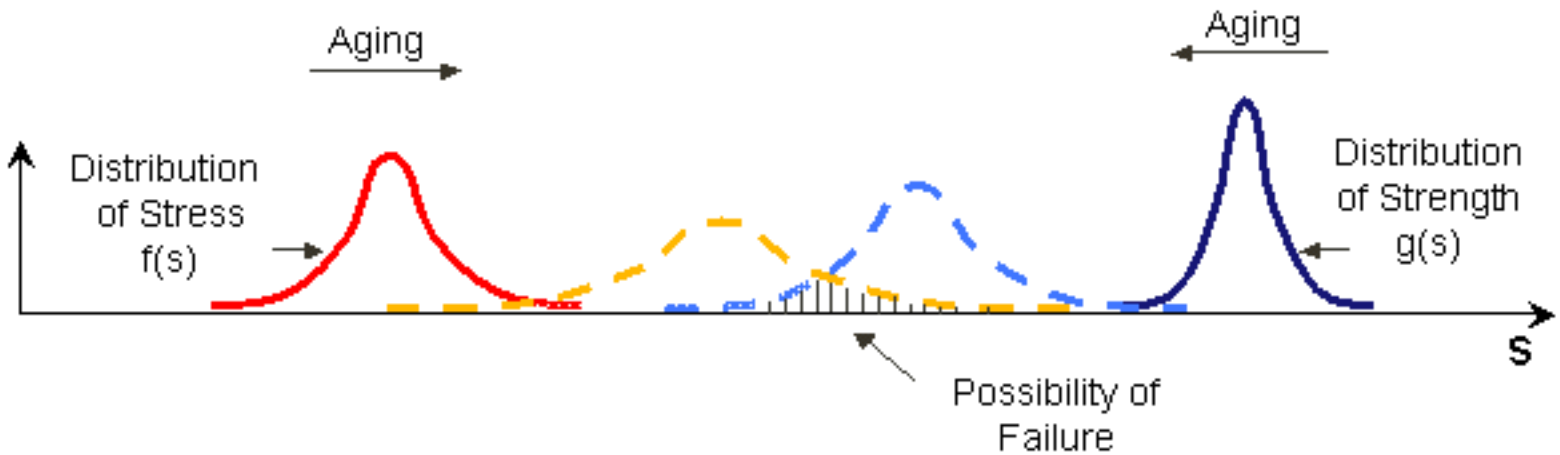
# Stress-Strength Interference Reliability Models




$$P(Stress > Strength) = \int_0^{\infty} \left\{ \int_0^{s} g(x)dx \right\} f(s)ds$$

Examples:

- Damage-Endurance Model
- Challenge-Tolerance Model
- Performance-Requirement Model



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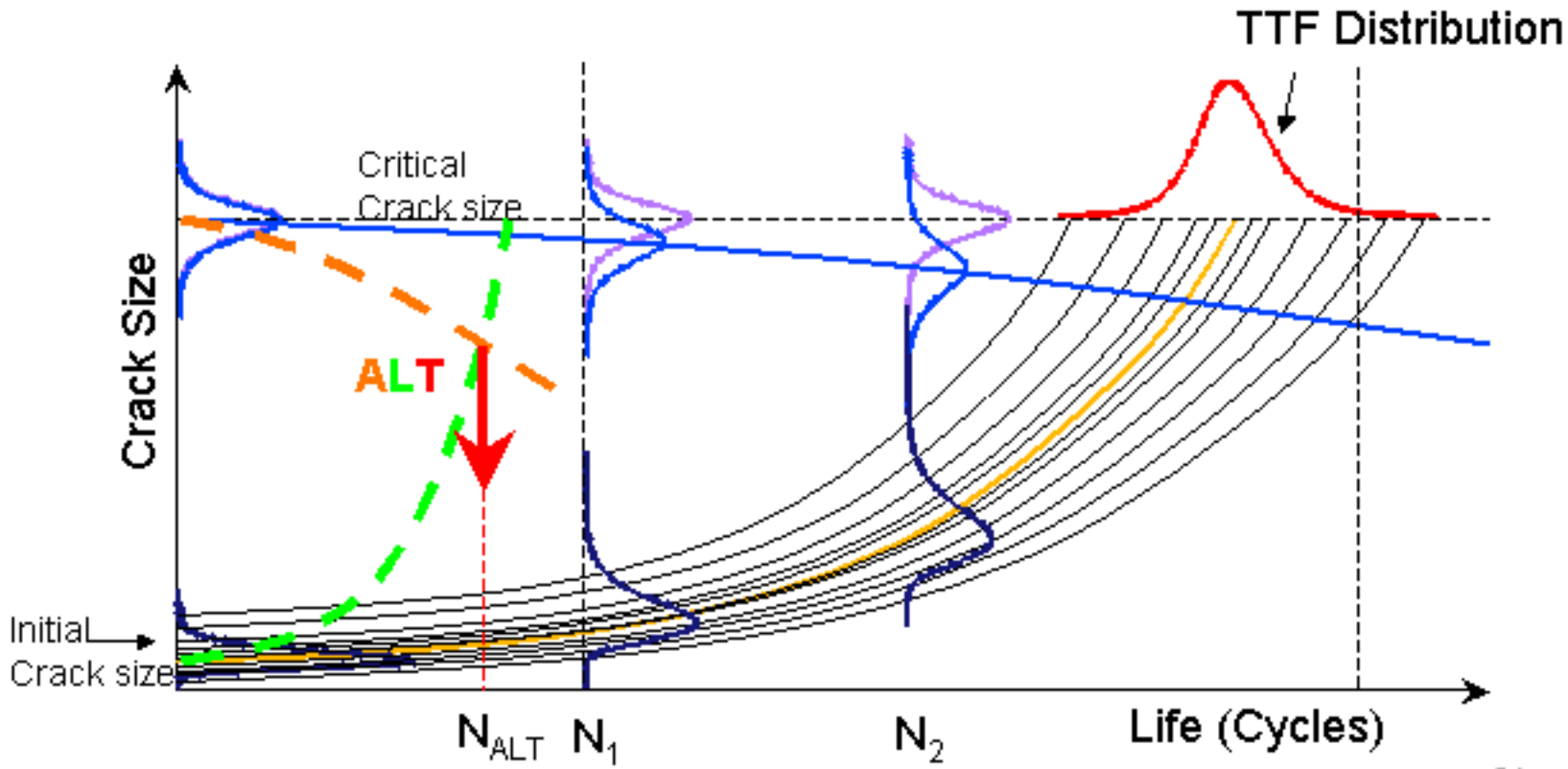
# Probabilistic Fracture Mechanics Approach to Fatigue



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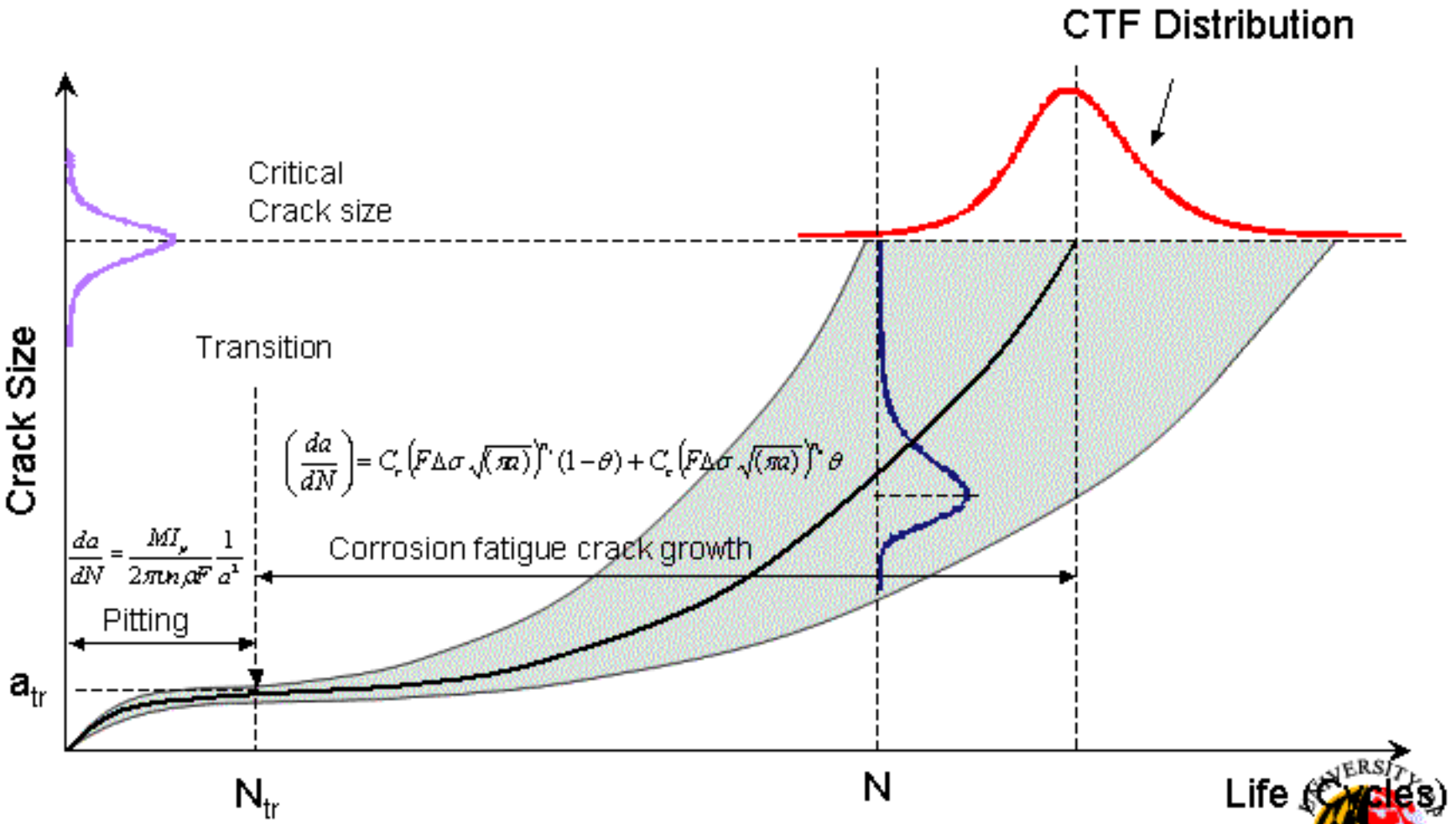
# Modeling Outline



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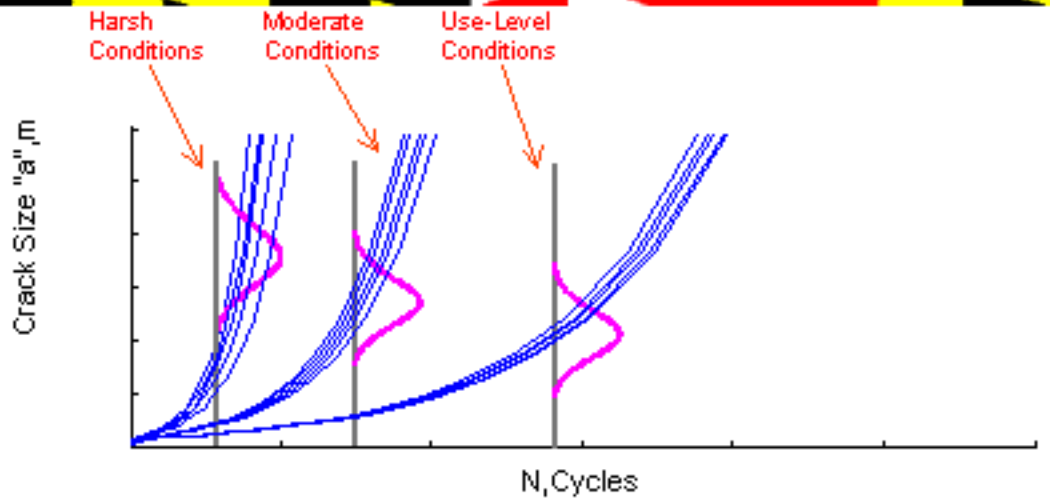
# The Proposed crack size vs. stress, cycle, etc.




$L(a_i) = LN(\mu_i, \sigma_i)$

$\mu_i = \ln[f(s_i, T_i, v_i, Ip_i, N_i)]$

$$a_i = \left[A \cdot s_i^{0.182} \cdot v_i^{-0.288} \cdot Ip_i^{0.248} \cdot N_i^{1/3} + B \cdot s_i^{3.24} \cdot v_i^{-0.377} \cdot Ip_i^{0.421} \cdot N_i^{2} \cdot e^{(4\times10^{-10} \cdot s_i^{2.062} \cdot v_i^{0.024} \cdot N_i)}\right]$$

*where a = crack size, v = load frequency, s = stress amplitude, $I_p$ = Current intensity, N = cycle No*

$$L(a) = f(a) = \frac{1}{\sigma \cdot a \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}\left(\ln a - \ln\left(A \cdot s^{0.182} \cdot v^{-0.288} \cdot Ip^{0.248} \cdot N^{1/3} + B \cdot s^{3.24} \cdot v^{-0.377} \cdot Ip^{0.421} \cdot N^{2} \cdot e^{(4\times10^{-10} \cdot s^{2.062} \cdot v^{0.024} \cdot N)}\right)\right)^2\right]$$



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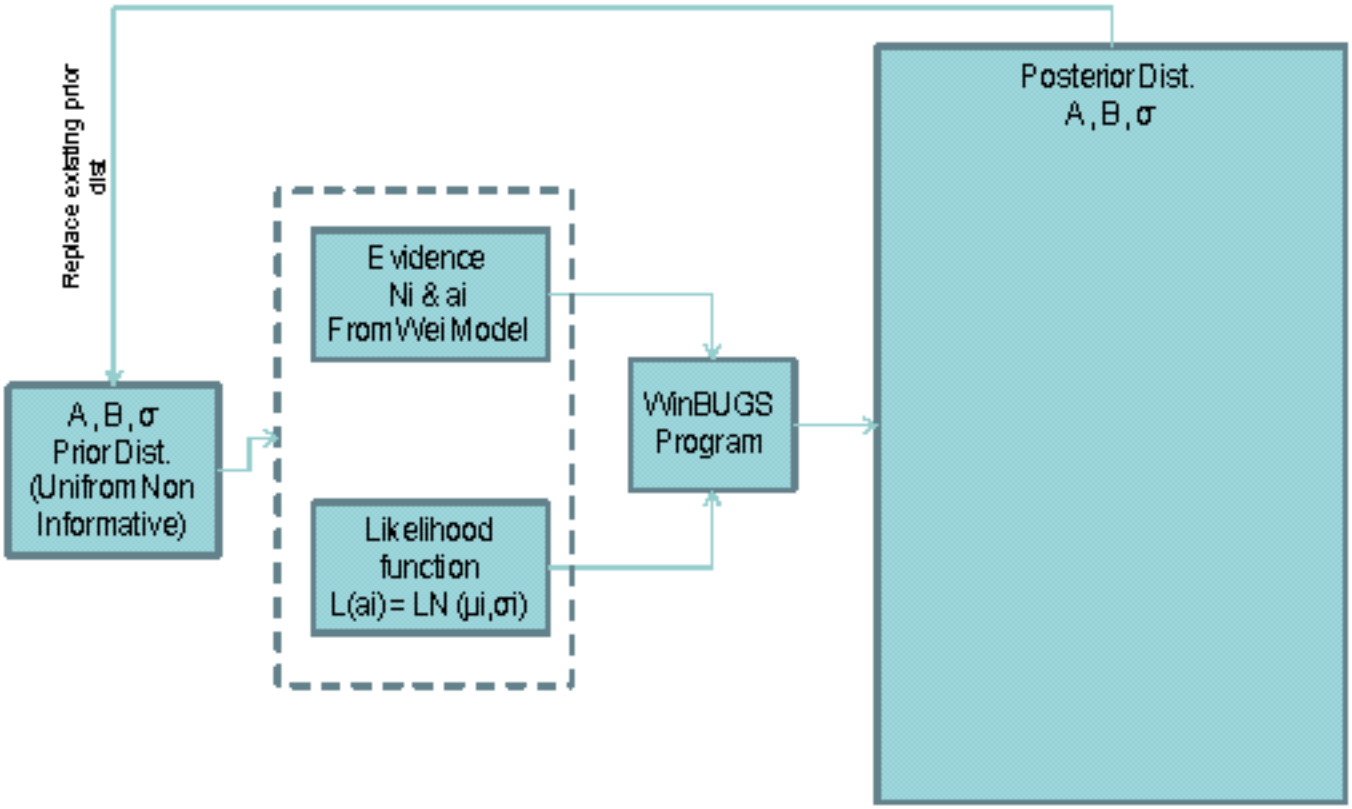
# The Bayesian approach

The Basic Algorithm for the WinBUGS Program to Estimate Distribution of Uncertain Parameters **A, B, and σ**.



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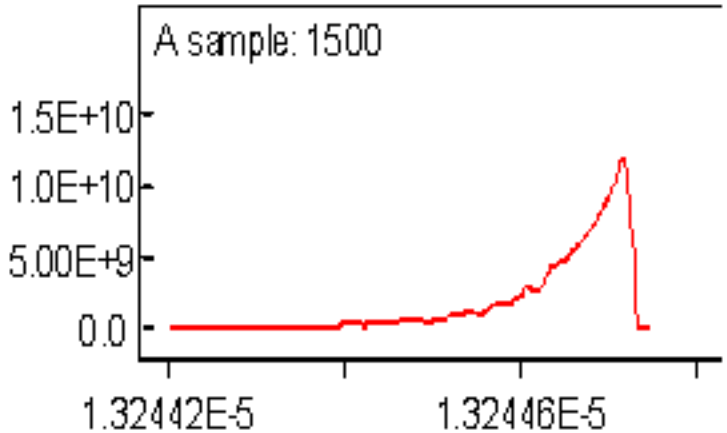
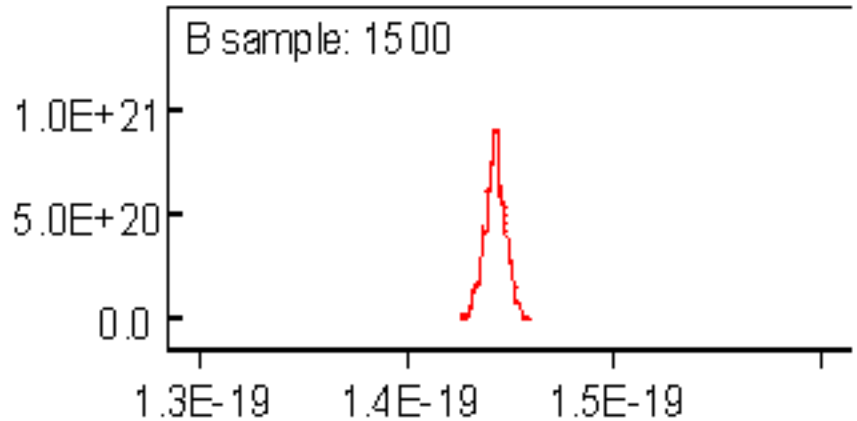
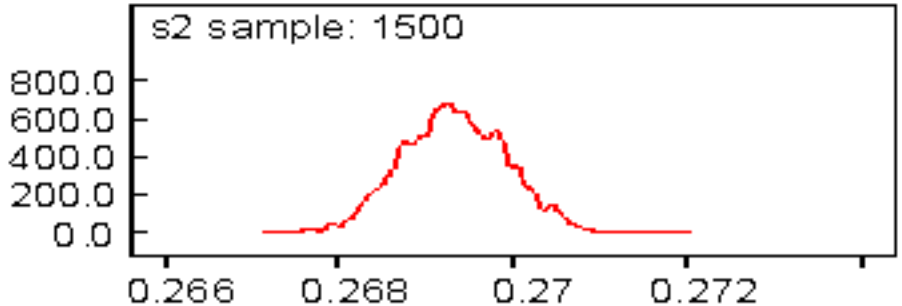
# The Bayesian Approach Results




A, B, and σ (s2) Posterior Distributions as Calculated in WinBUGS using test results for Carbon Steel X70



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# Example 2

# Model Uncertainty Assessment: Complex Fire Model Codes



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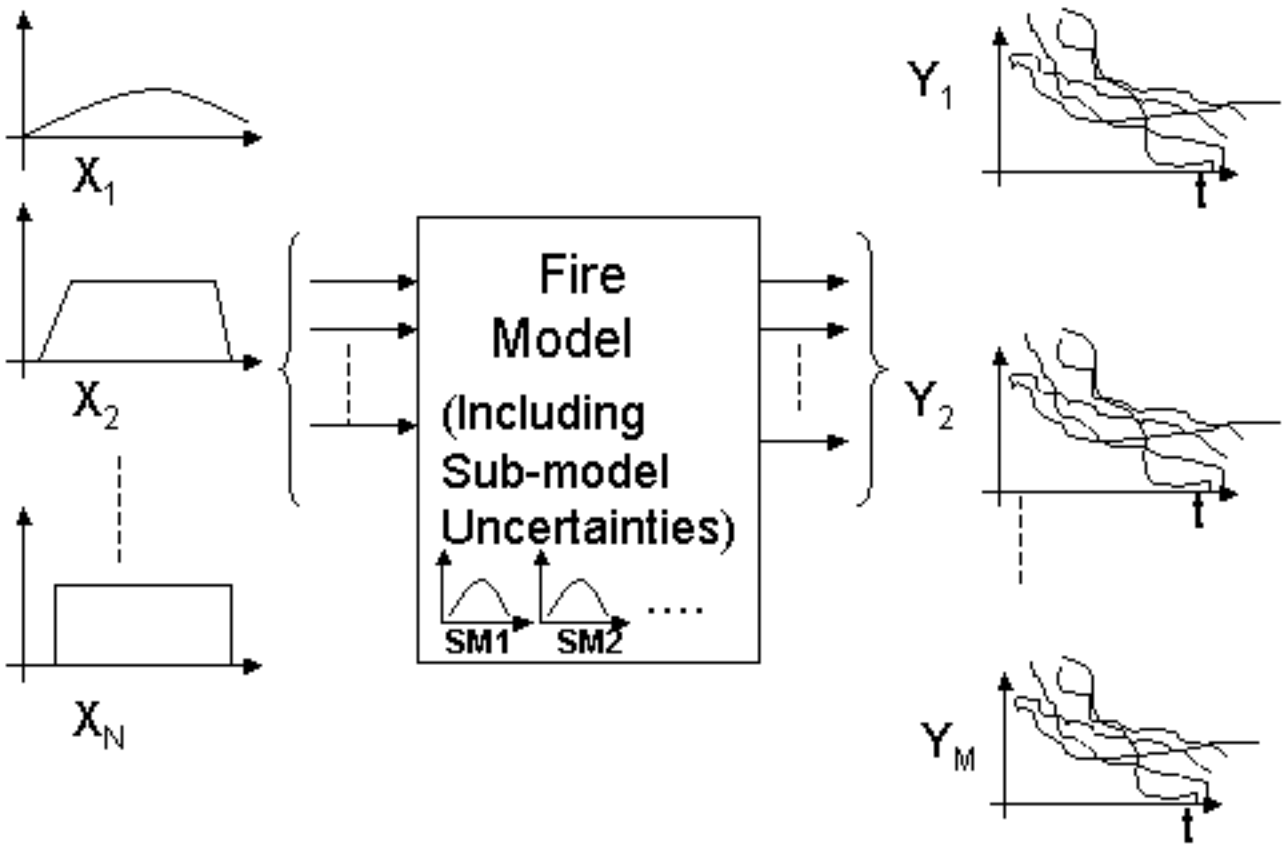
# Complex Model Output Uncertainty



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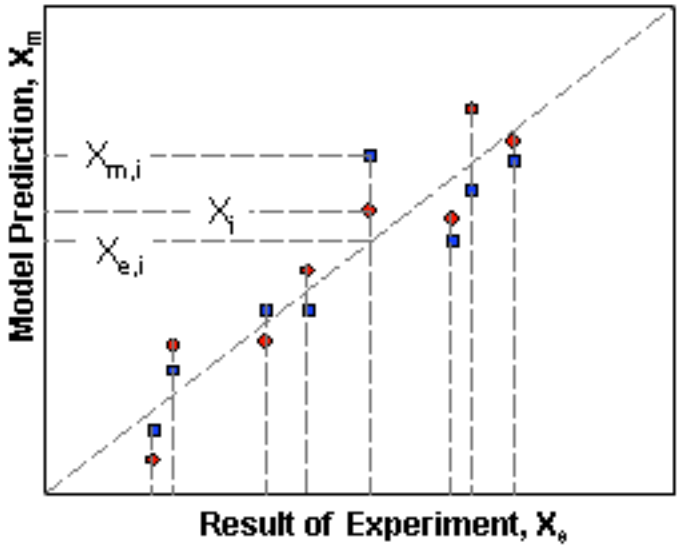
# Multiplicative Error Model




$$\frac{X_i}{X_{e,i}} = F_{e,i} \quad ; \quad F_e \sim LN(b_e, \sigma_e) \qquad (1)$$

$$\frac{X_i}{X_{m,i}} = F_{m,i} \quad ; \quad F_m \sim LN(b_m, \sigma_m) \qquad (2)$$

*where*:

$X$: Real Quantity

$X_e$: Result of experiment

$X_m$: Model prediction

$F_e$: The error factor for experimental data

$F_m$: The error factor for model predictions

$b_e, \sigma_e$: Mean and SD of experimental error factor

$b_m, \sigma_m$: Mean and SD of model error factor

Substituting (1) in (2) :

$$\left.\begin{array}{l} F_{e,i} X_{e,i} = F_{m,i} X_{m,i} \\ \dfrac{X_{e,i}}{X_{m,i}} = \dfrac{F_{m,i}}{F_{e,i}} = F_{t,i} \\ \text{Independency of } F_m, F_e \end{array}\right\} \Rightarrow F_t \sim LN\left(b_m - b_e, \sqrt{\sigma_m^2 + \sigma_e^2}\right)$$



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# Multiplicative Error: Bayesian Posterior

$$f(b_m,\sigma_m \mid X_{e,i},X_{m,i},b_e,\sigma_e) = \frac{f_0(b_m,\sigma_m)\times L(X_{e,i},X_{m,i},b_e,\sigma_e \mid b_m,\sigma_m)}{\int_{\sigma_m}\int_{b_m} f_0(b_m,\sigma_m)\times L(X_{e,i},X_{m,i},b_e,\sigma_e \mid b_m,\sigma_m)\,db_m\,d\sigma_m}$$

*where* :

$$L(X_{e,i},X_{m,i},b_e,\sigma_e \mid b_m,\sigma_m) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\left(\frac{X_{e,i}}{X_{m,i}}\right)\sqrt{\sigma_m^2+\sigma_e^2}}\, e^{-\frac{1}{2}\times\frac{\left[\ln\left(\frac{X_{e,i}}{X_{m,i}}\right)-(b_m-b_e)\right]^2}{\sigma_m^2+\sigma_e^2}}$$

$f_0(b_m,\sigma_m)$ : Prior Joint Distirbution of Parameters

$f(b_m,\sigma_m \mid X_{e,i},X_{m,i},b_e,\sigma_e)$ : Posterior Joint Distirbution of Parameters

Given a model prediction such as $X_m$ the distribution of the real value X will be :

$$\left.\begin{array}{l} X_m \text{ given as model prediction} \\ F_m \sim LN(b_m,\sigma_m) \\ X = F_m X_m \end{array}\right\} \Rightarrow X \sim LN(\ln(X_m)+b_m,\sigma_m)$$



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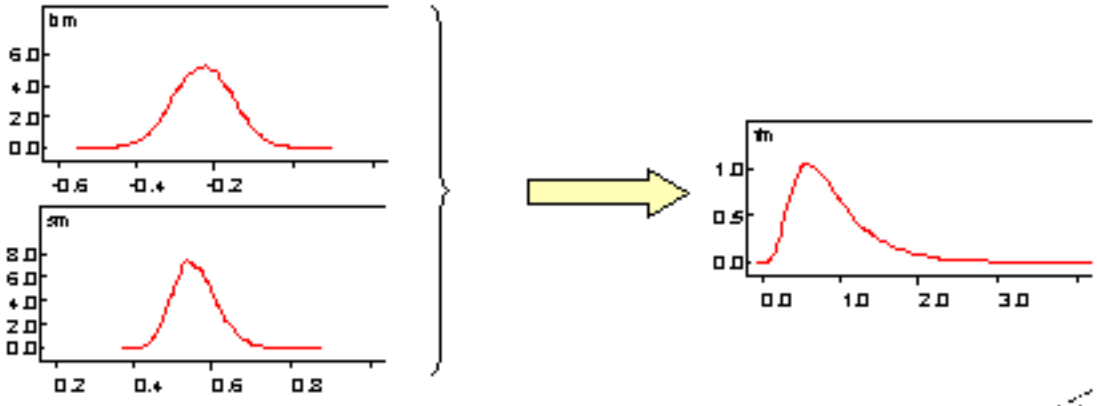
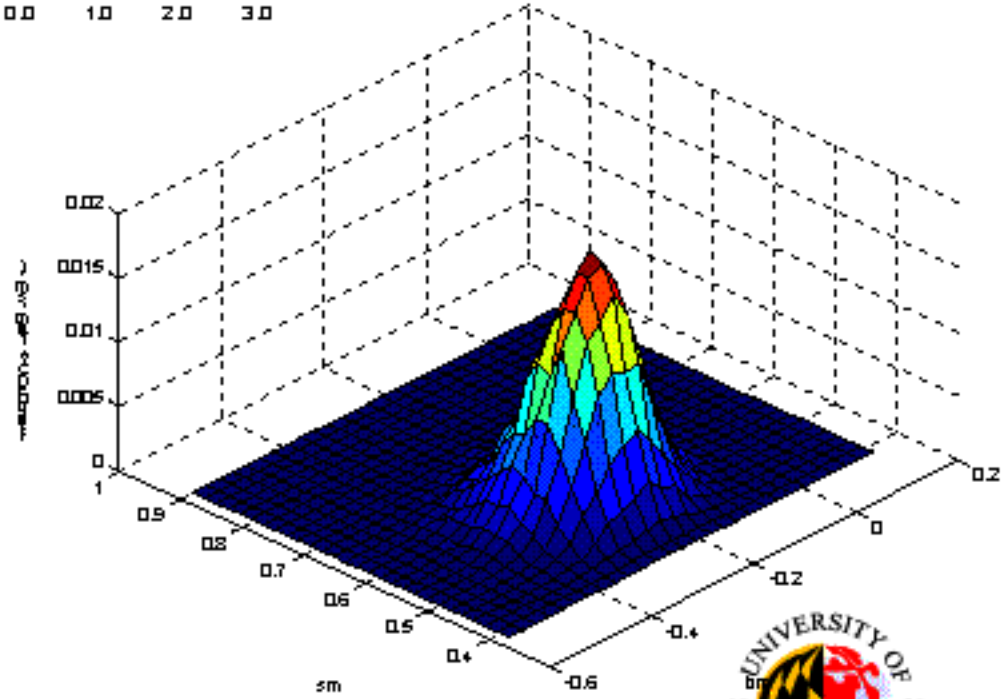
# Radiant Heat Flux Predicted by the FIVE Model Using WinBugs




| node | mean | sd | MC error | 2.50% | median | 97.50% |
|---|---|---|---|---|---|---|
| $b_m$ | -0.2297 | 0.07582 | 5.26E-04 | -0.3789 | -0.2297 | -0.08211 |
| $f_m$ | 0.934 | 0.5818 | 0.003578 | 0.2627 | 0.7962 | 2.417 |
| $s_m$ | 0.5579 | 0.05698 | 4.16E-04 | 0.4603 | 0.5533 | 0.6799 |
| y[1] | 0.934 | 0.5818 | 0.003578 | 0.2627 | 0.7962 | 2.417 |
| y[5] | 4.67 | 2.909 | 0.01789 | 1.314 | 3.981 | 12.08 |
| y[10] | 9.34 | 5.818 | 0.03578 | 2.627 | 7.962 | 24.17 |
| y[15] | 14.01 | 8.727 | 0.05367 | 3.941 | 11.94 | 36.25 |
| y[20] | 18.68 | 11.64 | 0.07155 | 5.255 | 15.92 | 48.33 |



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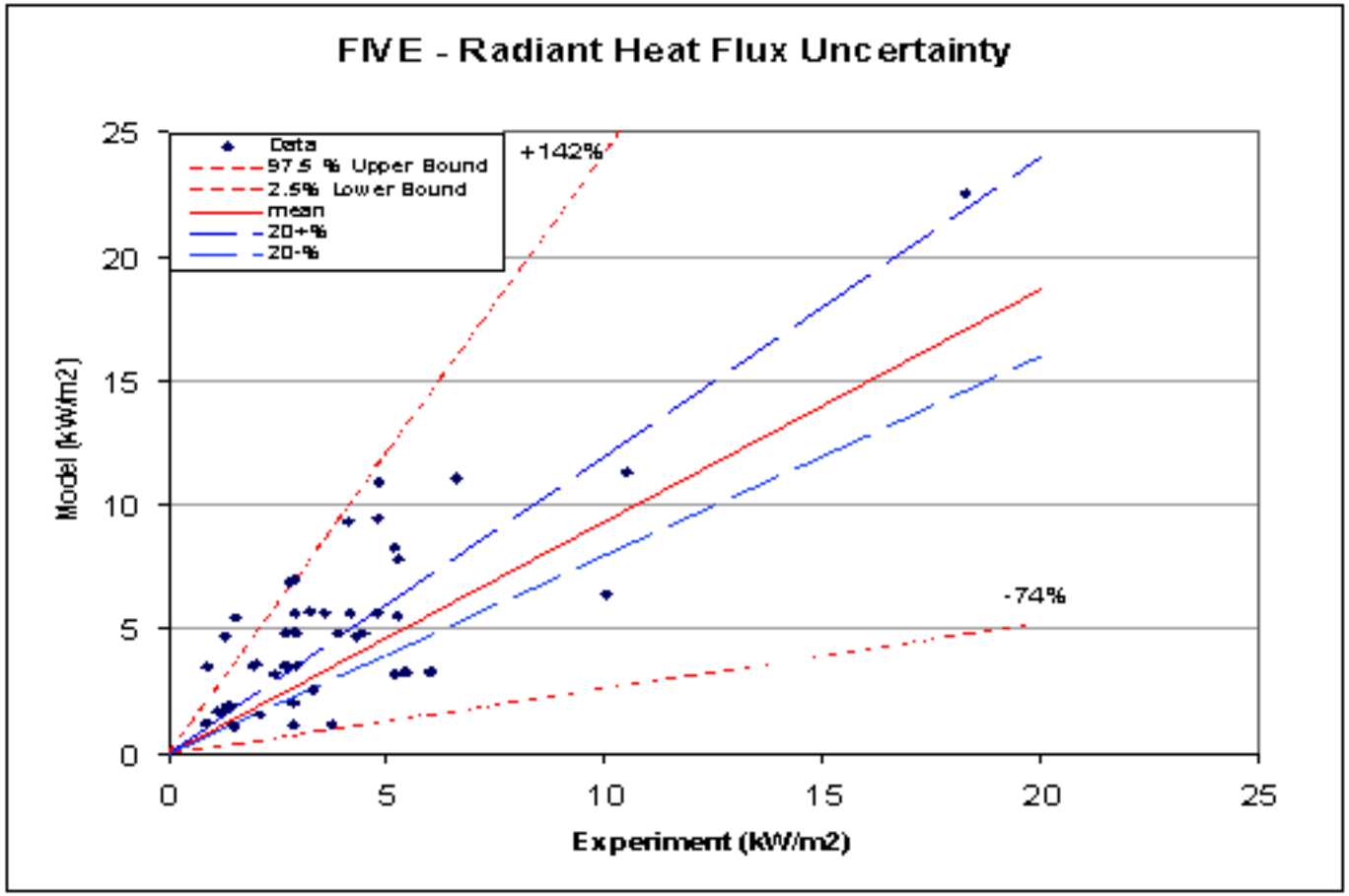
# Uncertainty in Radiant Heat Flux Predicted by the FIVE Model



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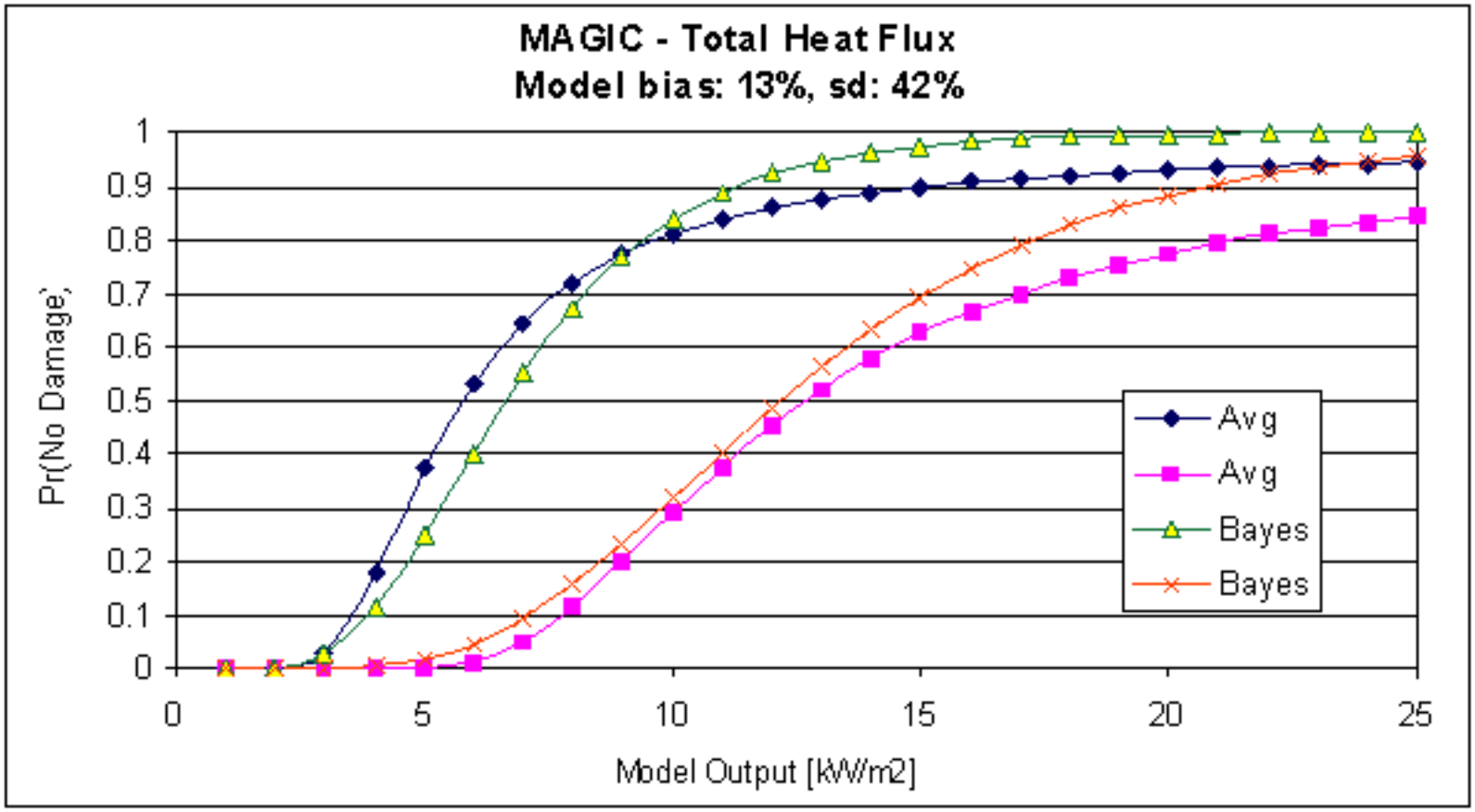
# Plots for Heat Flux (Cont.)



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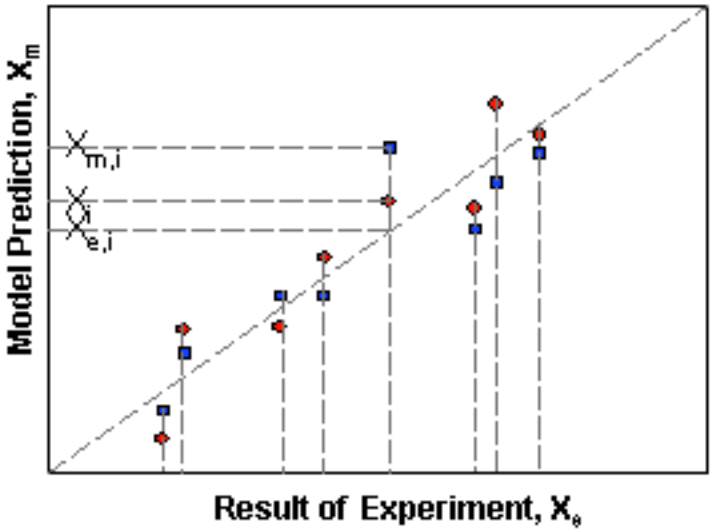
# Another Option: Percentage Error Model




$$\frac{X_i - X_{e,i}}{X_{e,i}} = E_{e,i} \quad ; \quad E_e \sim N(b_e, \sigma_e) \qquad (1)$$

$$\frac{X_i - X_{m,i}}{X_{m,i}} = E_{m,i} \quad ; \quad E_m \sim N(b_m, \sigma_m) \qquad (2)$$

$$\frac{X_{e,i} - X_{m,i}}{X_{m,i}} = E_{em,i} \;, (1),(2) \Rightarrow E_{em} = \frac{E_m - E_e}{E_e + 1} \qquad (3)$$

*where* :

$X$ : Real Quantity

$X_e$ : Result of experiment

$X_m$ : Model prediction

$b_e, \sigma_e$ : Mean and SD of experimental percentage error

$b_m, \sigma_m$ : Mean and SD of model percentage error

$b_{em}, \sigma_{em}$ : Mean and SD percentage error of Experiment compared to model

$$\left.\begin{array}{l}\text{Having Equation (3) \&} \\ \text{Independency of } E_m, E_e\end{array}\right\} \Rightarrow E_{em} \sim N\left(\frac{b_m - b_e}{1+b_e} + \frac{\sigma_e^2(1+b_m)}{(1+b_e)^3}, \sqrt{\frac{\sigma_m^2}{(1+b_e)^2} + \frac{\sigma_e^2(1+b_m)^2}{(1+b_e)^4}}\right)$$



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# Example 3

# Probabilistic Fatigue Failure Model Development with Diverse Type of Data



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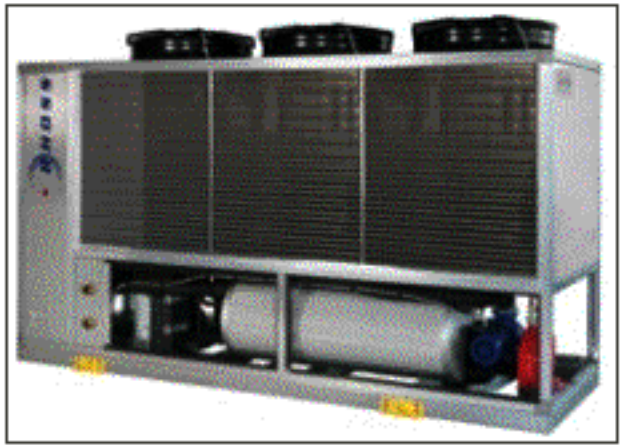
# CASE STUDY: FATIGUE LIFE

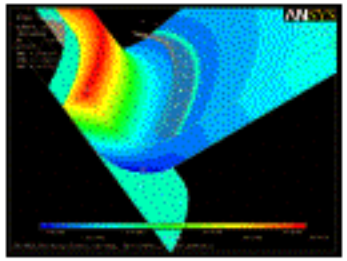


AC Unit

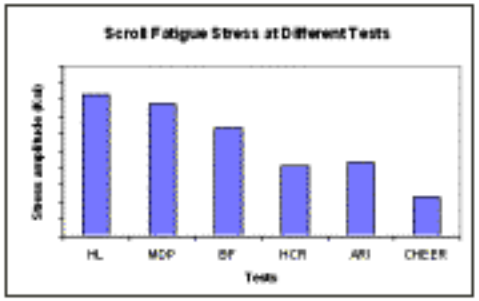
Compressor

Scrolls

Modeling



FEA




Stress at Different Tests



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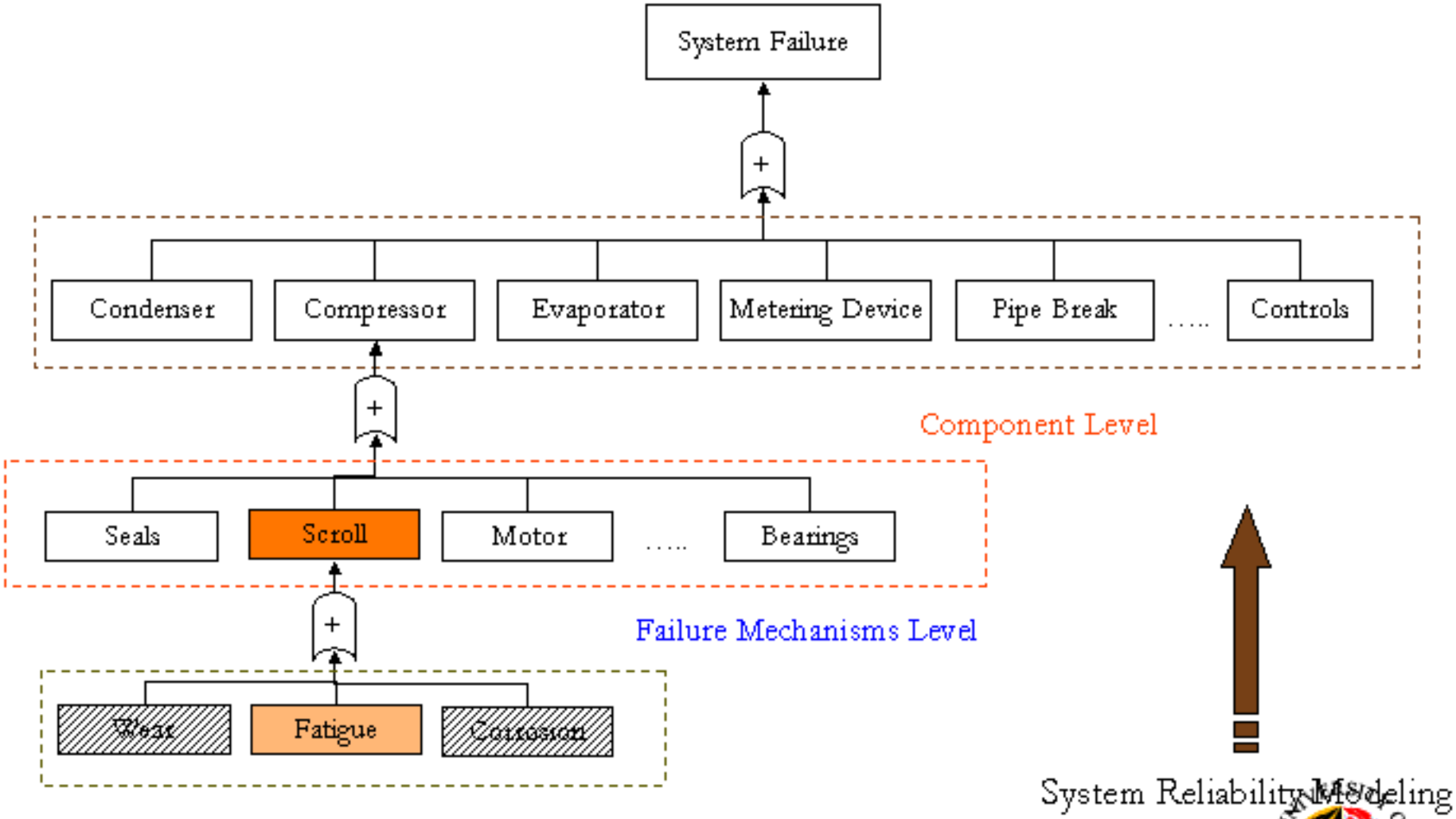
# RELIABILITY MODEL OF THE SYSTEM



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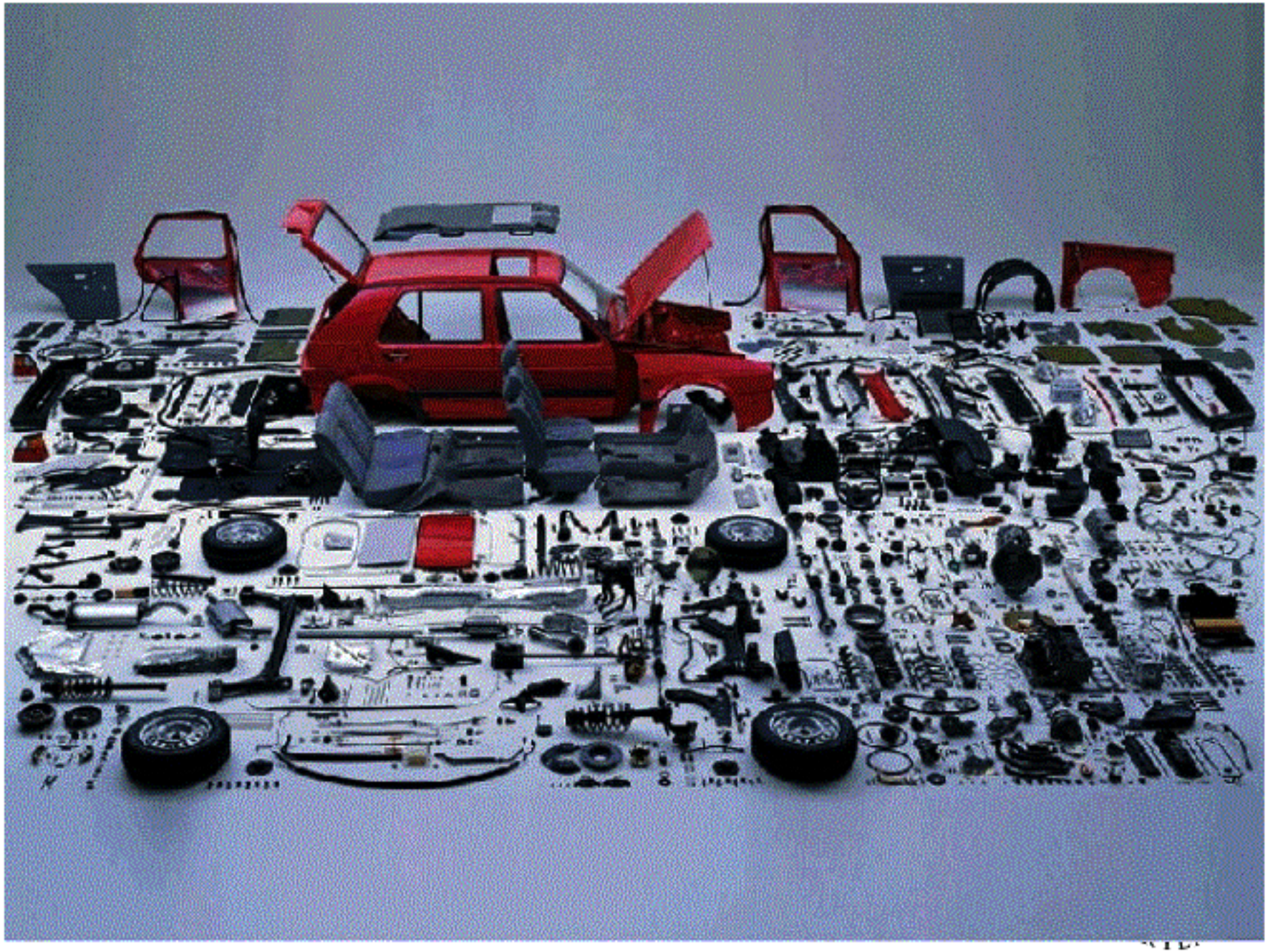

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# MODELING MATHEMATICS

- Consider Lognormal as Distribution of Time to failure:

$$f(t) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{[\ln(t)-\ln(\mu)]^2}{2\sigma^2}}$$

$\mu$ = Median
$\sigma$ = Standard Deviation
$t$ = Time to Failure

- Inverse Power Law relates median life ($\mu = N_{median}$) to Stress (S)

$$\mu = \frac{1}{KS^n}$$

$\mu$ = Median
$K, n$ = Constants to be determined from data analysis
$S$ = Stress

- Final Life Model

$$f(t \mid S, K, n, \sigma) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{[\ln(t)+\ln(K)+n\ln(S)]^2}{2\sigma^2}}$$



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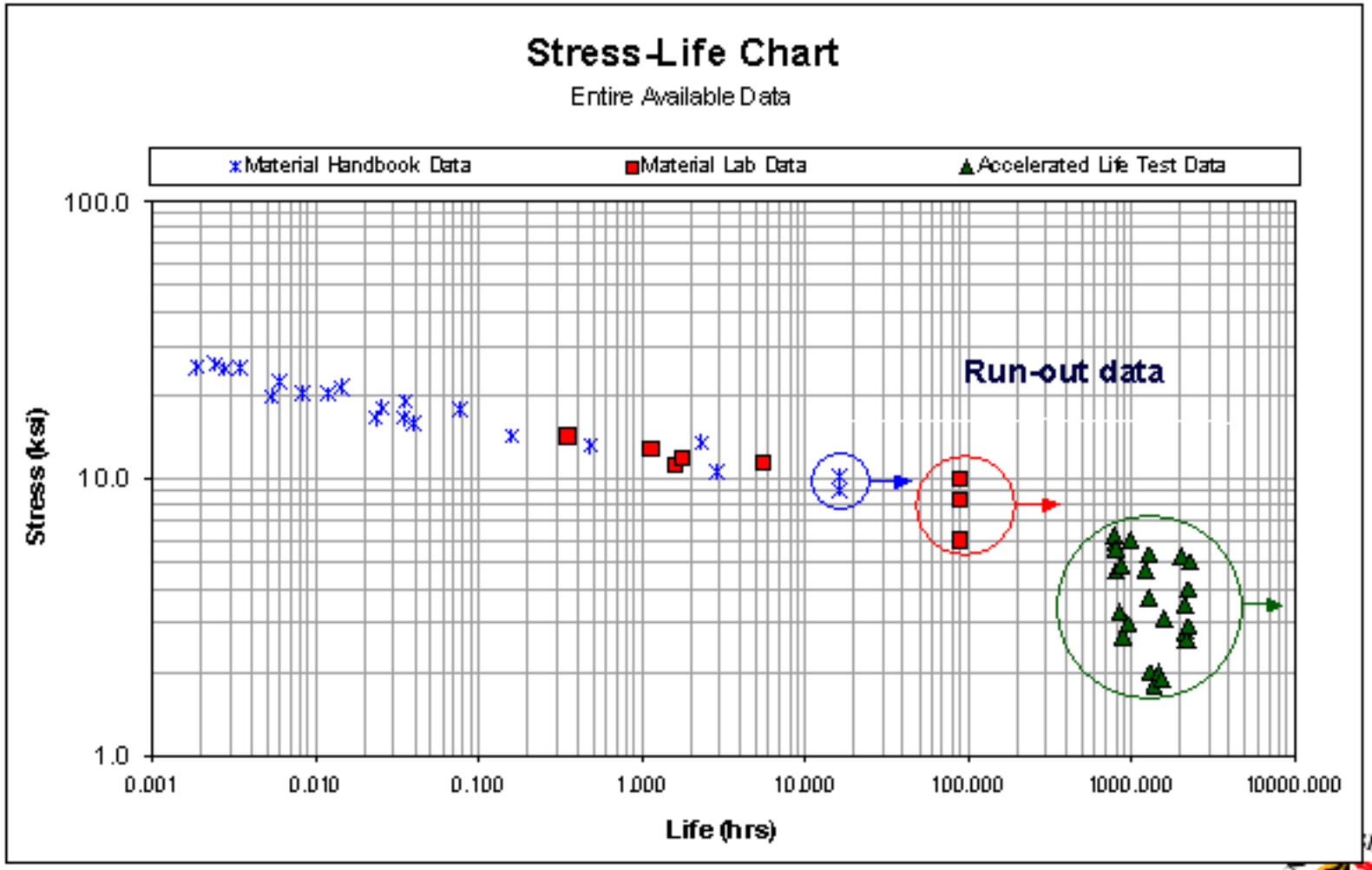
# AVAILABLE DATA



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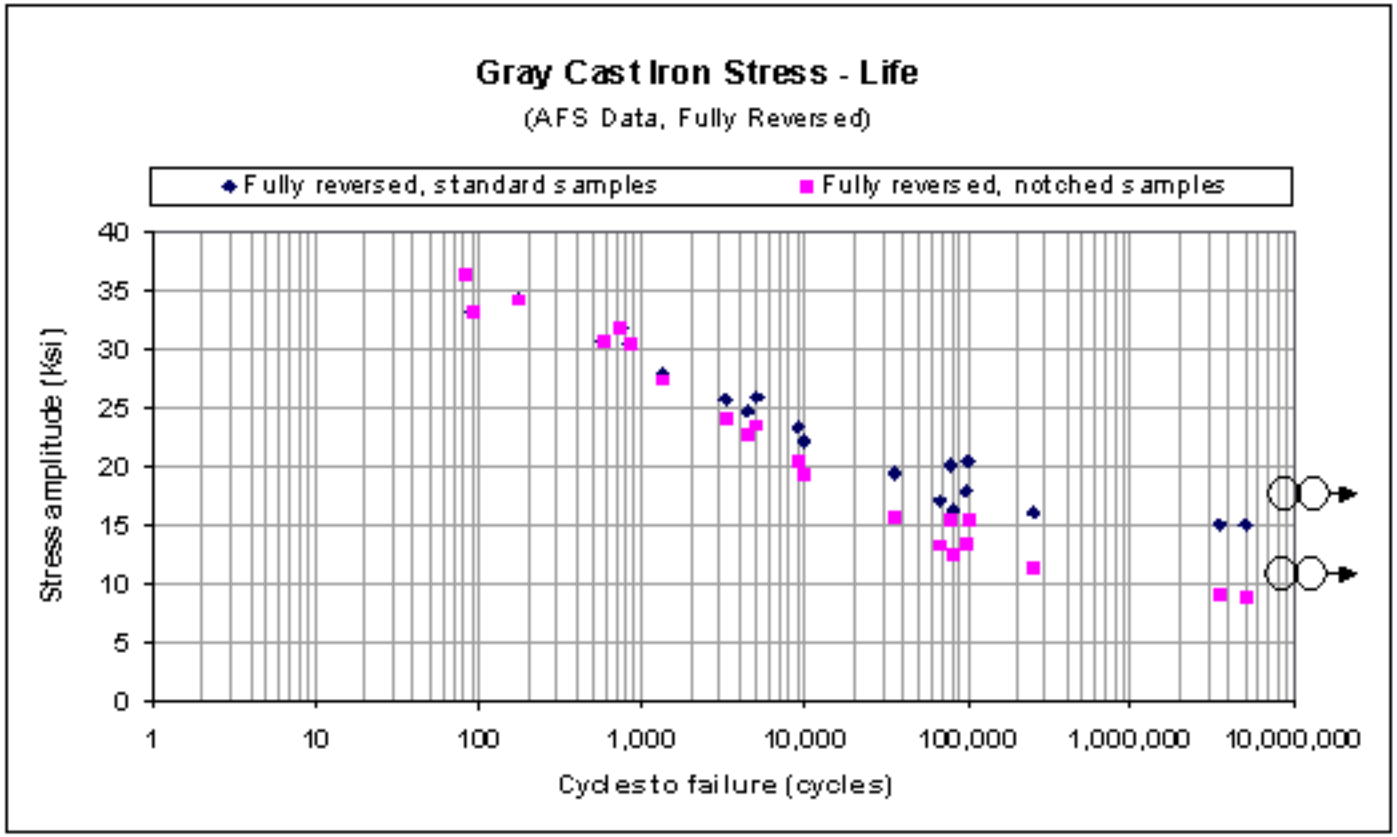
# Gray Cast Iron Fatigue Data

## American Foundry Society, ASTM 35B



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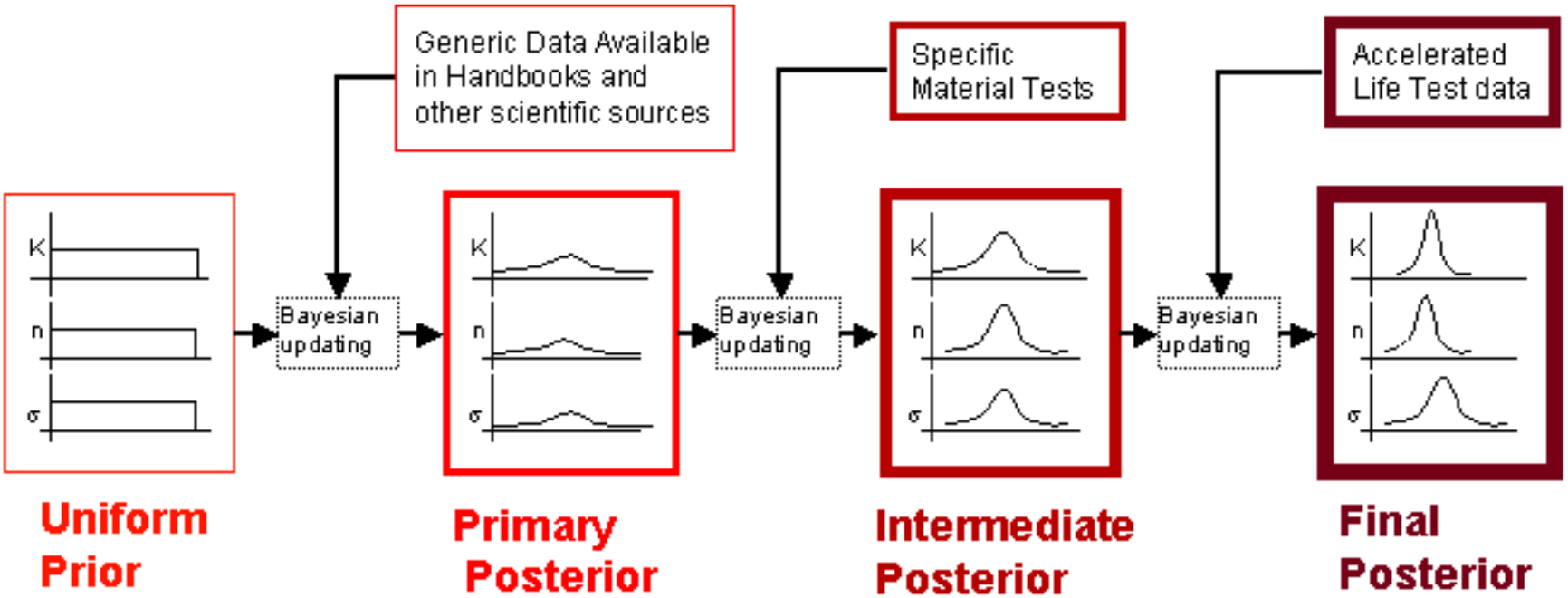
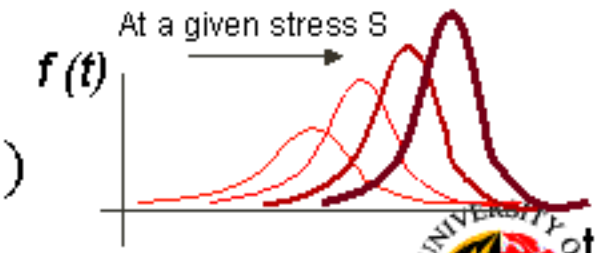
# A SEQUENTIAL BAYESIAN APPROACH OUTLINE




**Stress-Life Distribution:** $f(t / S, K, n, \sigma)$

Where K, n and σ are parameters



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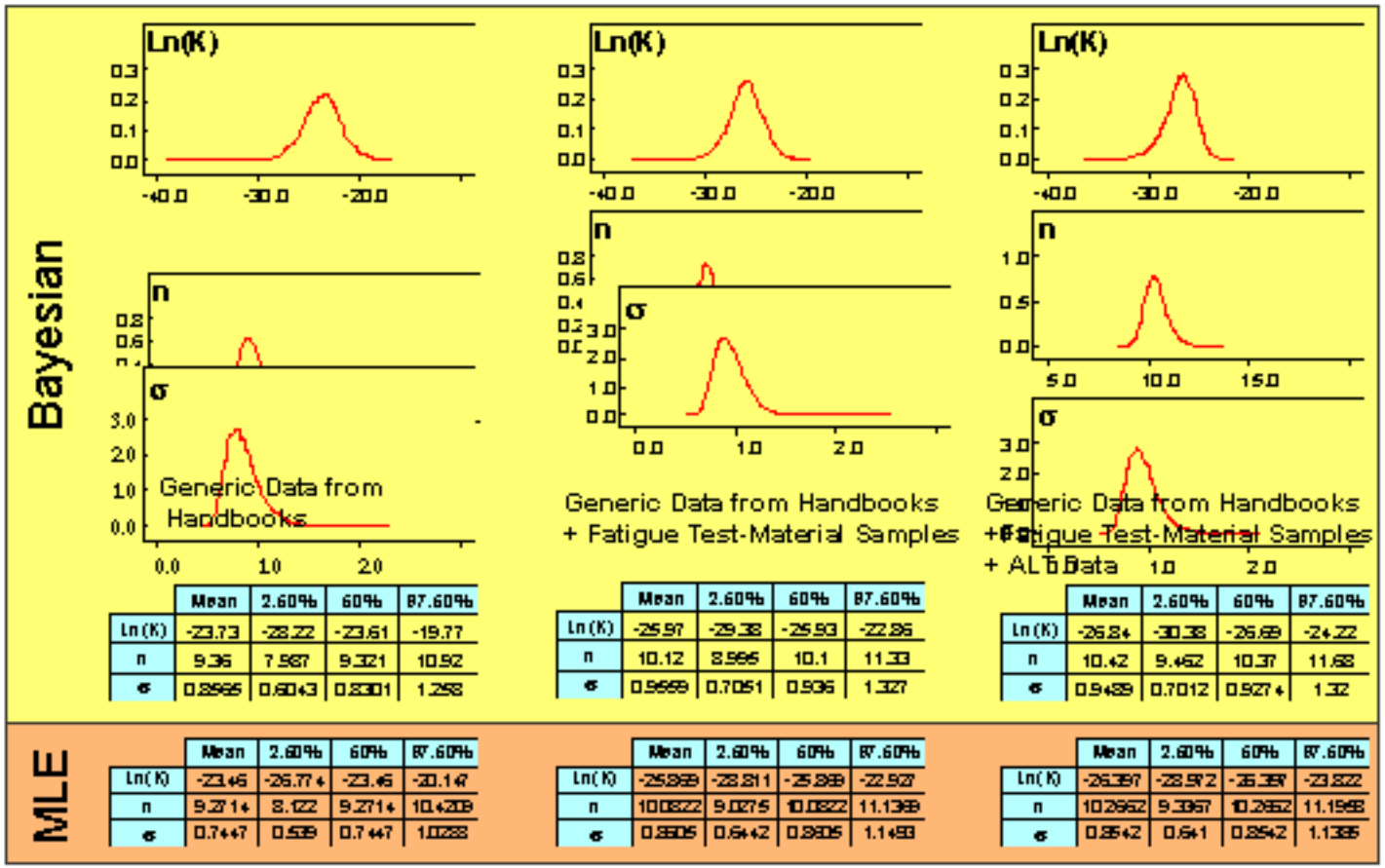
# EVOLUTION OF THE MODEL PARAMETERS




Bayesian

| | Mean | 2.60% | 60% | 97.60% |
|---|---|---|---|---|
| Ln(K) | -23.73 | -28.22 | -23.61 | -19.77 |
| n | 9.36 | 7.587 | 9.321 | 10.92 |
| σ | 0.8565 | 0.6043 | 0.8301 | 1.258 |

| | Mean | 2.60% | 60% | 97.60% |
|---|---|---|---|---|
| Ln(K) | -25.97 | -29.38 | -25.93 | -22.86 |
| n | 10.12 | 8.566 | 10.1 | 11.33 |
| σ | 0.9659 | 0.7051 | 0.936 | 1.327 |

| | Mean | 2.60% | 60% | 97.60% |
|---|---|---|---|---|
| Ln(K) | -26.84 | -30.38 | -26.69 | -24.22 |
| n | 10.42 | 9.452 | 10.37 | 11.68 |
| σ | 0.9489 | 0.7012 | 0.9274 | 1.32 |

MLE

| | Mean | 2.60% | 60% | 97.60% |
|---|---|---|---|---|
| Ln(K) | -23.46 | -26.774 | -23.46 | -20.147 |
| n | 9.2714 | 8.122 | 9.2714 | 10.4209 |
| σ | 0.7447 | 0.539 | 0.7447 | 1.0288 |

| | Mean | 2.60% | 60% | 97.60% |
|---|---|---|---|---|
| Ln(K) | -25.989 | -28.811 | -25.989 | -22.927 |
| n | 10.0822 | 9.0275 | 10.0822 | 11.1369 |
| σ | 0.8605 | 0.6442 | 0.8605 | 1.1493 |

| | Mean | 2.60% | 60% | 97.60% |
|---|---|---|---|---|
| Ln(K) | -26.397 | -28.972 | -26.397 | -23.822 |
| n | 10.2962 | 9.3967 | 10.2962 | 11.1958 |
| σ | 0.8542 | 0.641 | 0.8542 | 1.1335 |



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# Example 4

# Assessment of Airframe Integrity Involving Fatigue Crack Model Assessment Based on Inspection and NDI Observations



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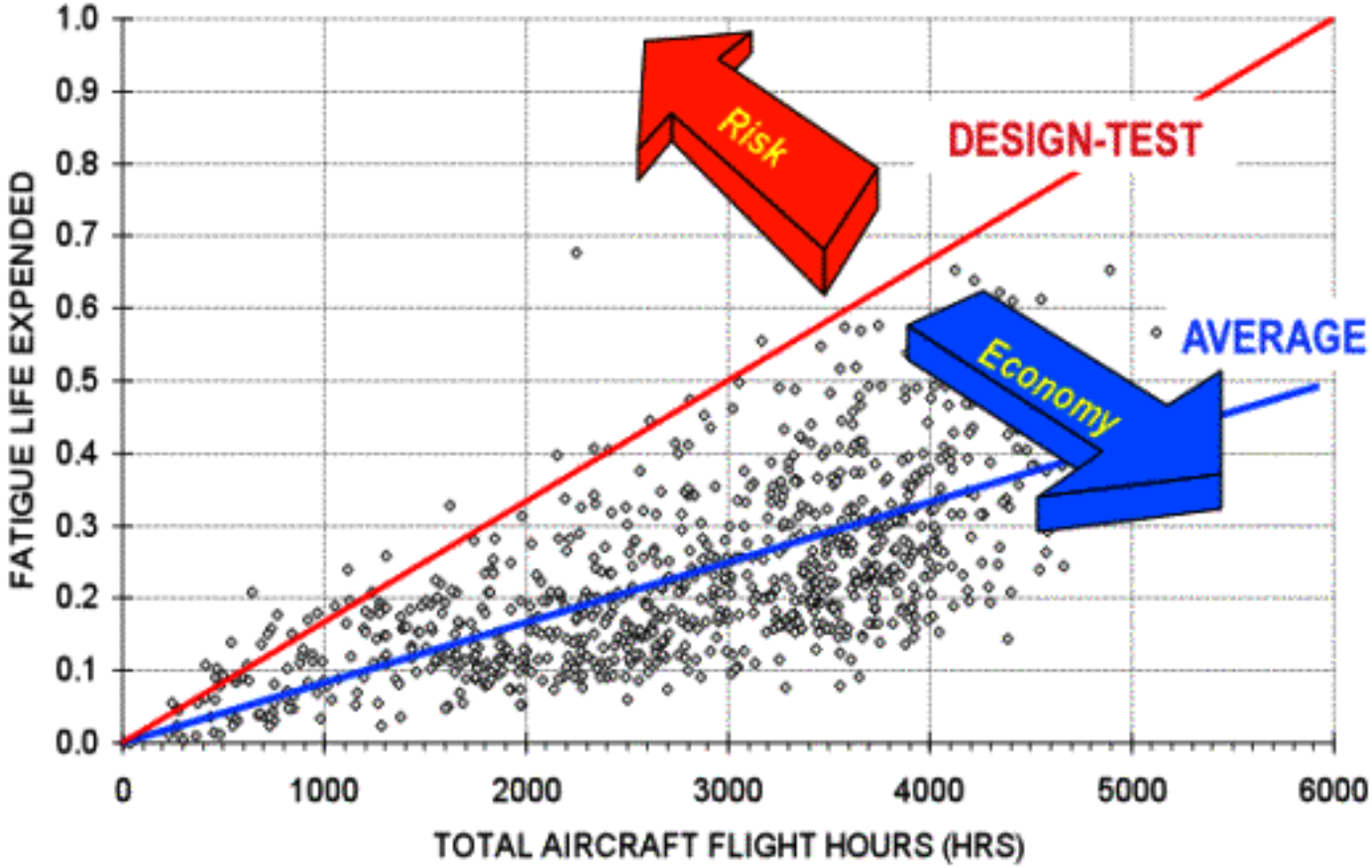
# Individual Aircraft FLE Scatter As Tracked

FLE=Flight Life Expended (a measure of Palmgren-Miner's cumulative damage)




FLE=1 or 100% = Miner's cumulative damage of 0.5



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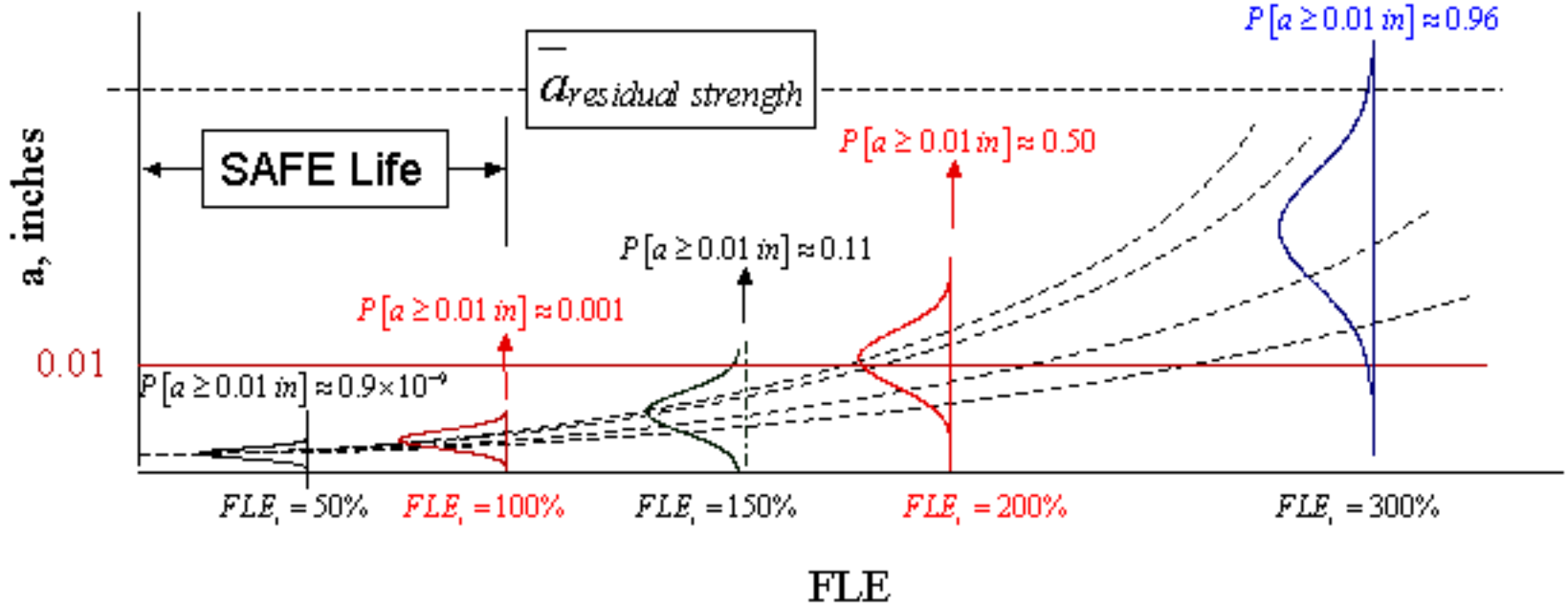
# Interpreting FLE in terms of Probability of a > 0.01 inches

**Note: 0.01 inches=0.25 mm << $a_{critical}$**




**Challenge: Quantify the probability of exceeding residual strength**



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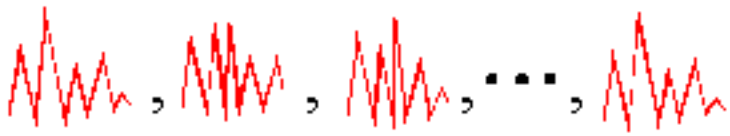
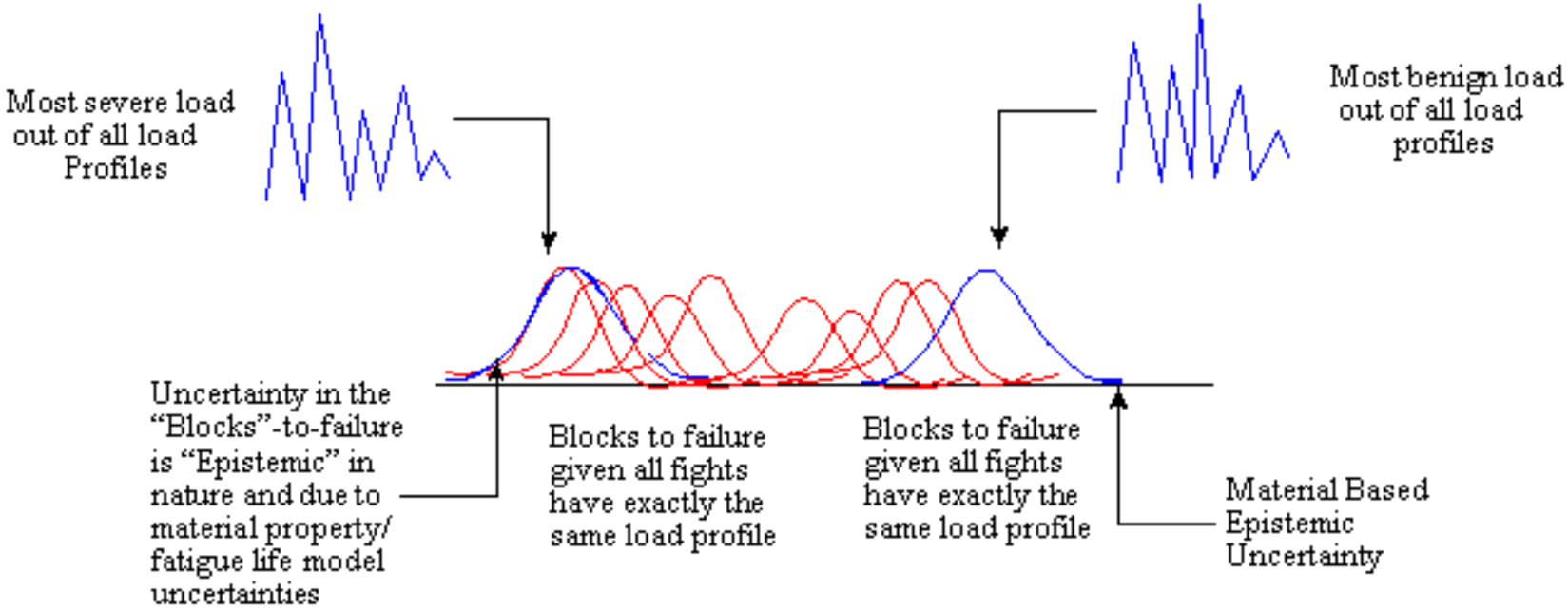
# Life Scatter versus Load Variability



158 load profiles ranked according to damage increment, Δ (Miner's Rule)




Note:
Epistemic uncertainties are reducible (by more tests, better models, etc.)



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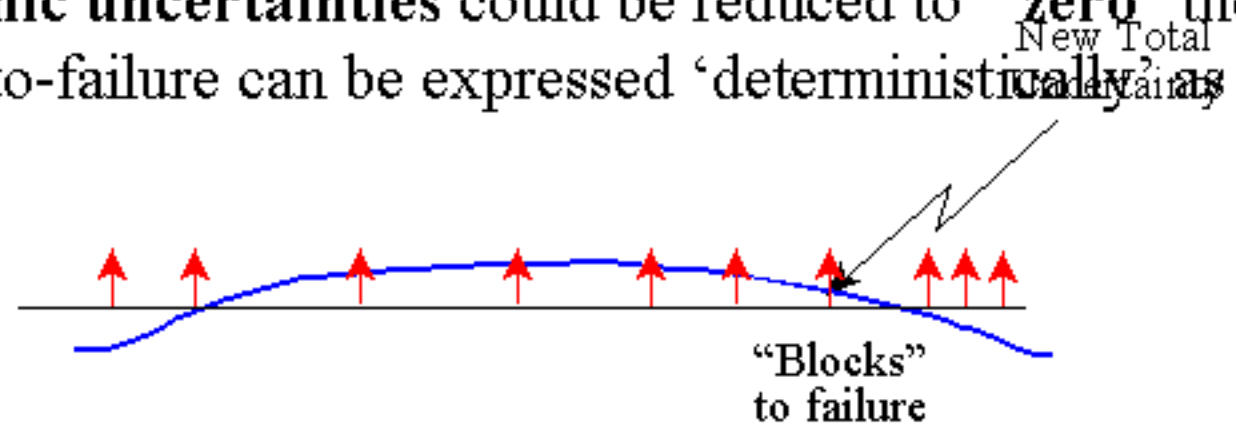
# Uncertainty is Certain!

If **epistemic uncertainties** could be reduced to "**zero**" the 'blocks'-to-failure can be expressed 'deterministically' as




The total uncertainty will not be significantly reduced by removing the Epistemic uncertainties. The total uncertainty is dominated by the Aleatory uncertainty (Load)

**Technical Challenge:**

To what degree should epistemic uncertainty be reduced? In other words, what is the point of diminishing return in reducing epistemic uncertainties (due to material, testing, modeling etc)?



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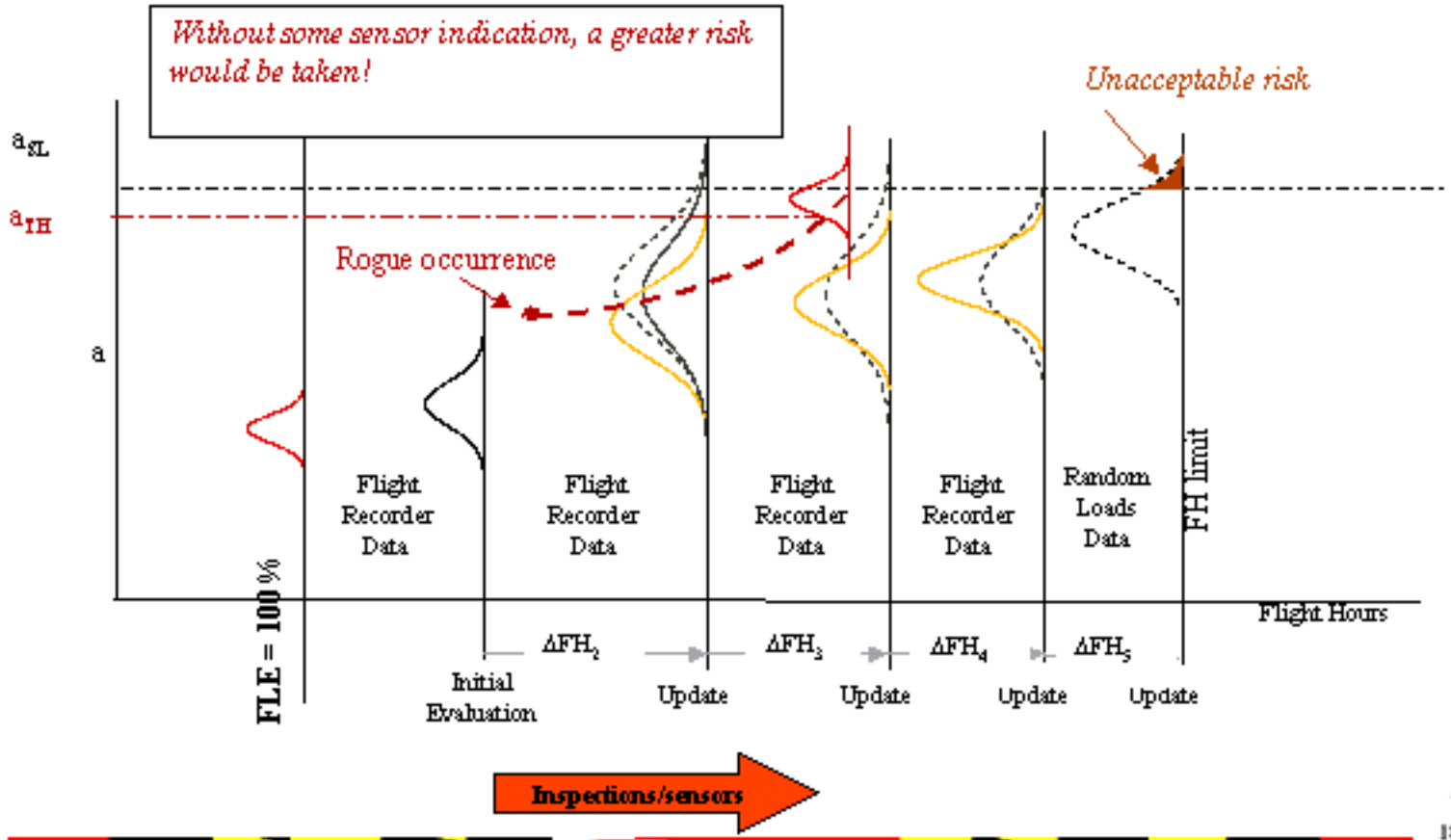
# NDI-BASED PROGNOSIS

To mitigate the **risk of a rogue flaw** a sensor suite and/or an inspection schedule would provide the feedback.



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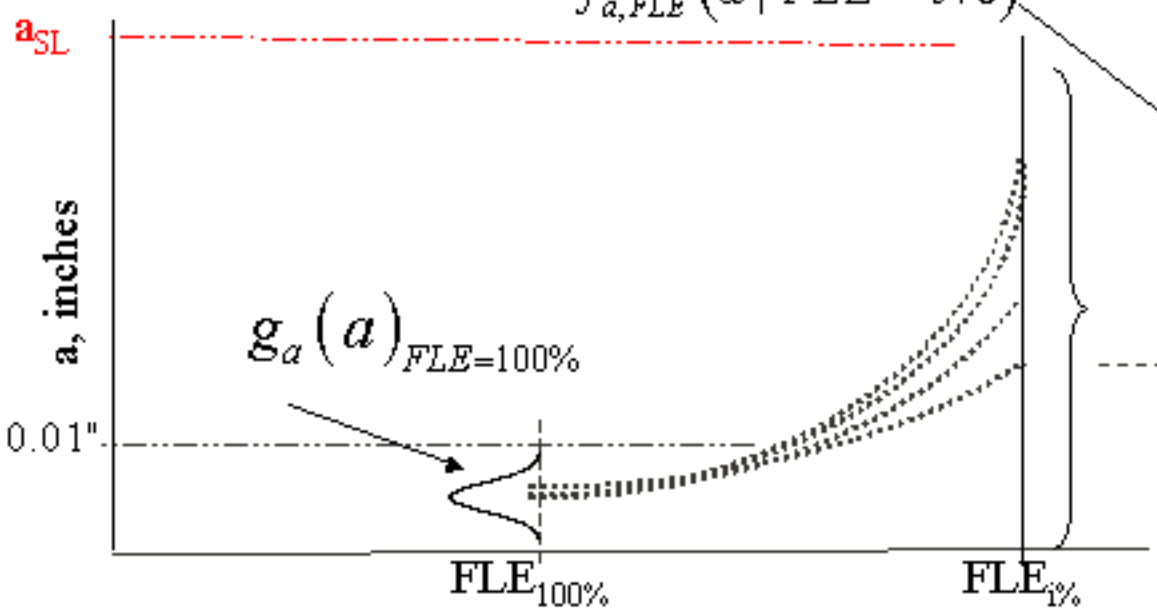
# Joint Probability Density Function of Crack Size & FLE

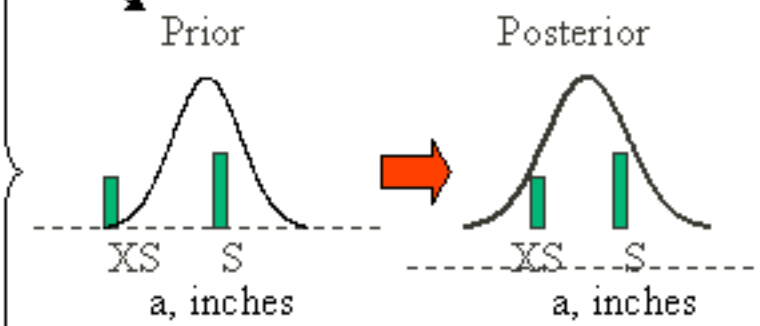
For an inspected aircraft at FLE=i%, generate random crack growth with know loads and random material properties.
Fit a joint probability density function.

$$f_{a,FLE}(a, FLE) = \frac{1}{a \cdot \sigma \cdot \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(a)+\ln(k)+n\cdot\ln\left(\frac{1}{FLE}\right)}{\sigma}\right)^2}$$

Changed parameters
k, n, & σ

$$f_{a,FLE}(a \mid FLE = i\%)$$



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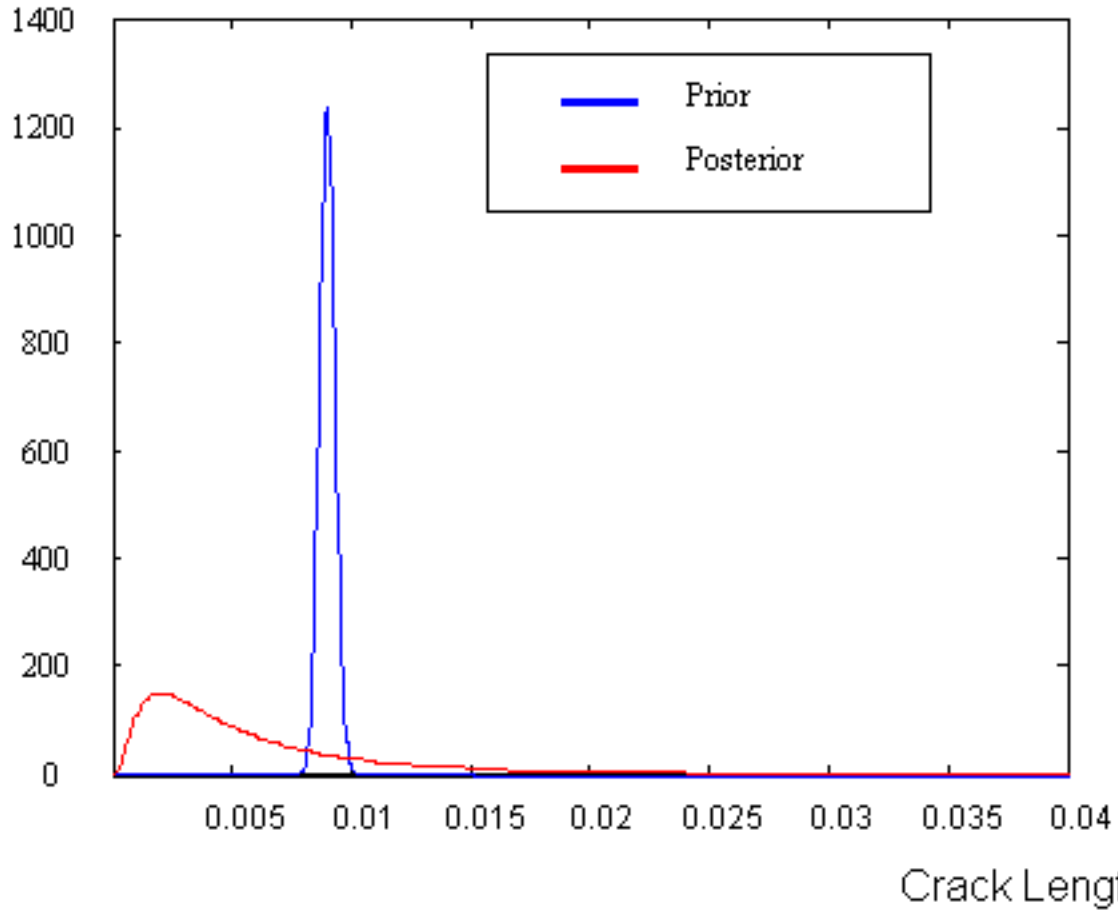
# Results for 40 Aircraft in a Fleet

Crack length Distribution At 100%FLE



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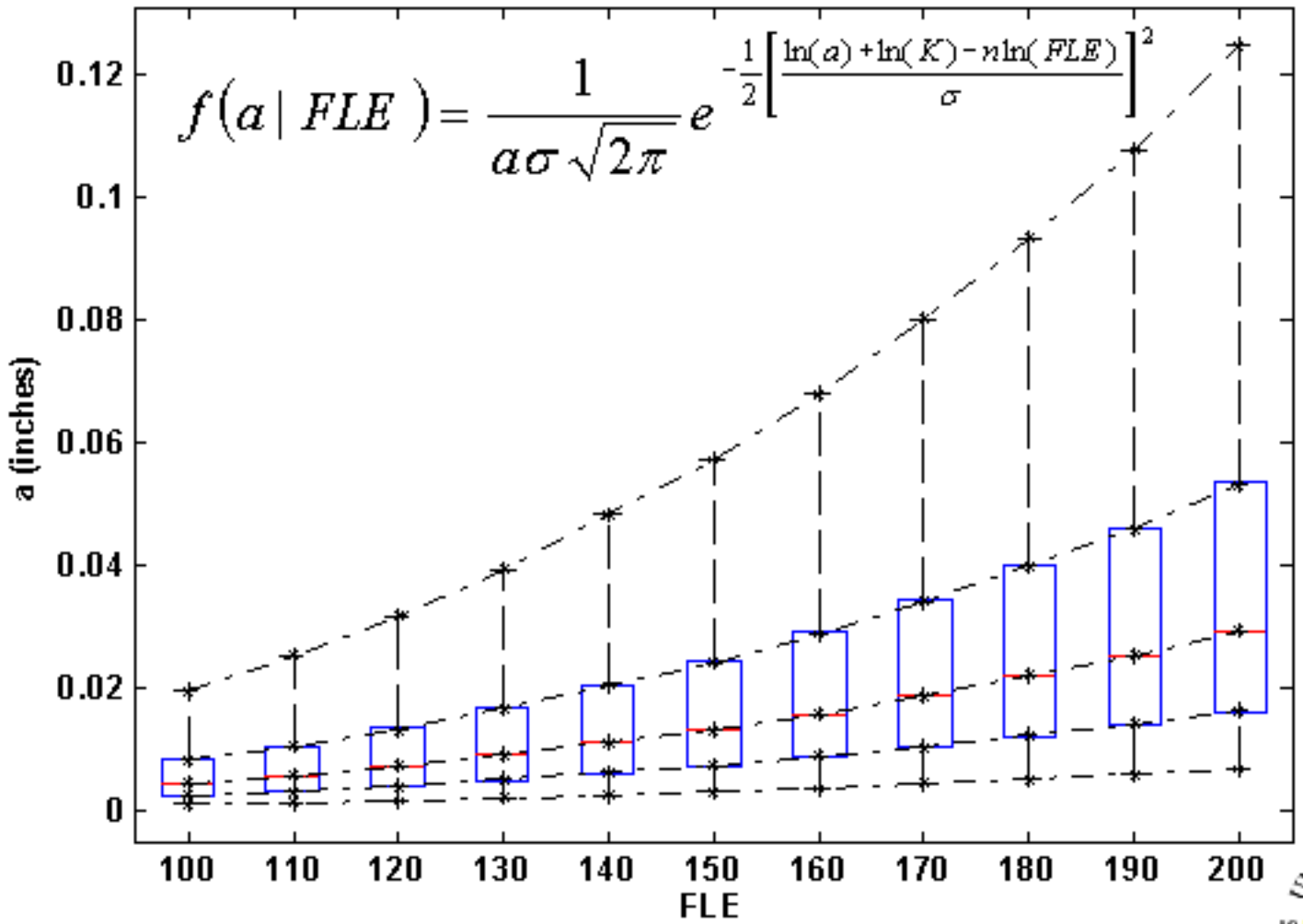
# Crack size & FLE Joint Density Function



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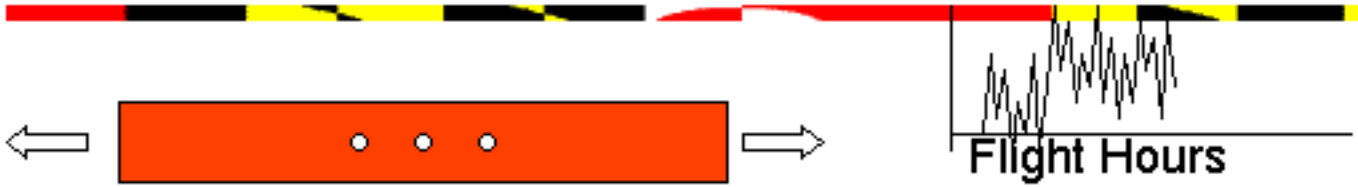
# Comparing to Experimental Data



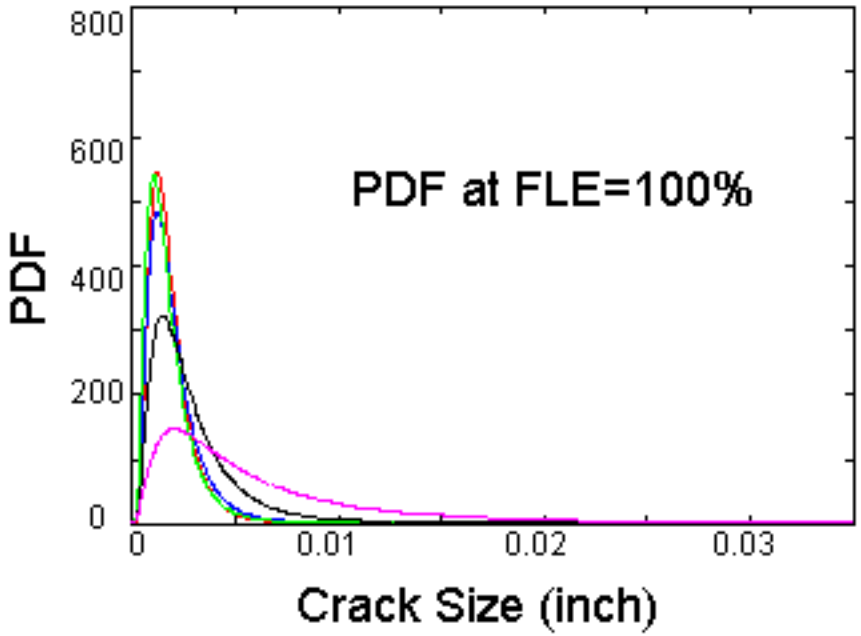

AA-7075, 3.0g +1.5σ

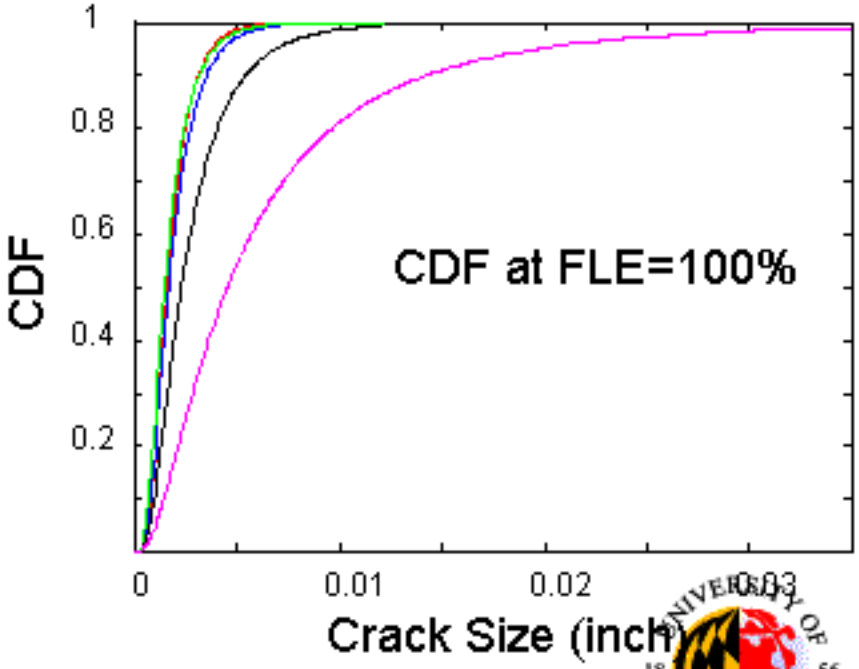
AA-7075, 4.5g+1.5σ

AA-7050, 3.0g+1.5σ

AA-7050, 4.5g+1.5σ

Fleet Findings — P3



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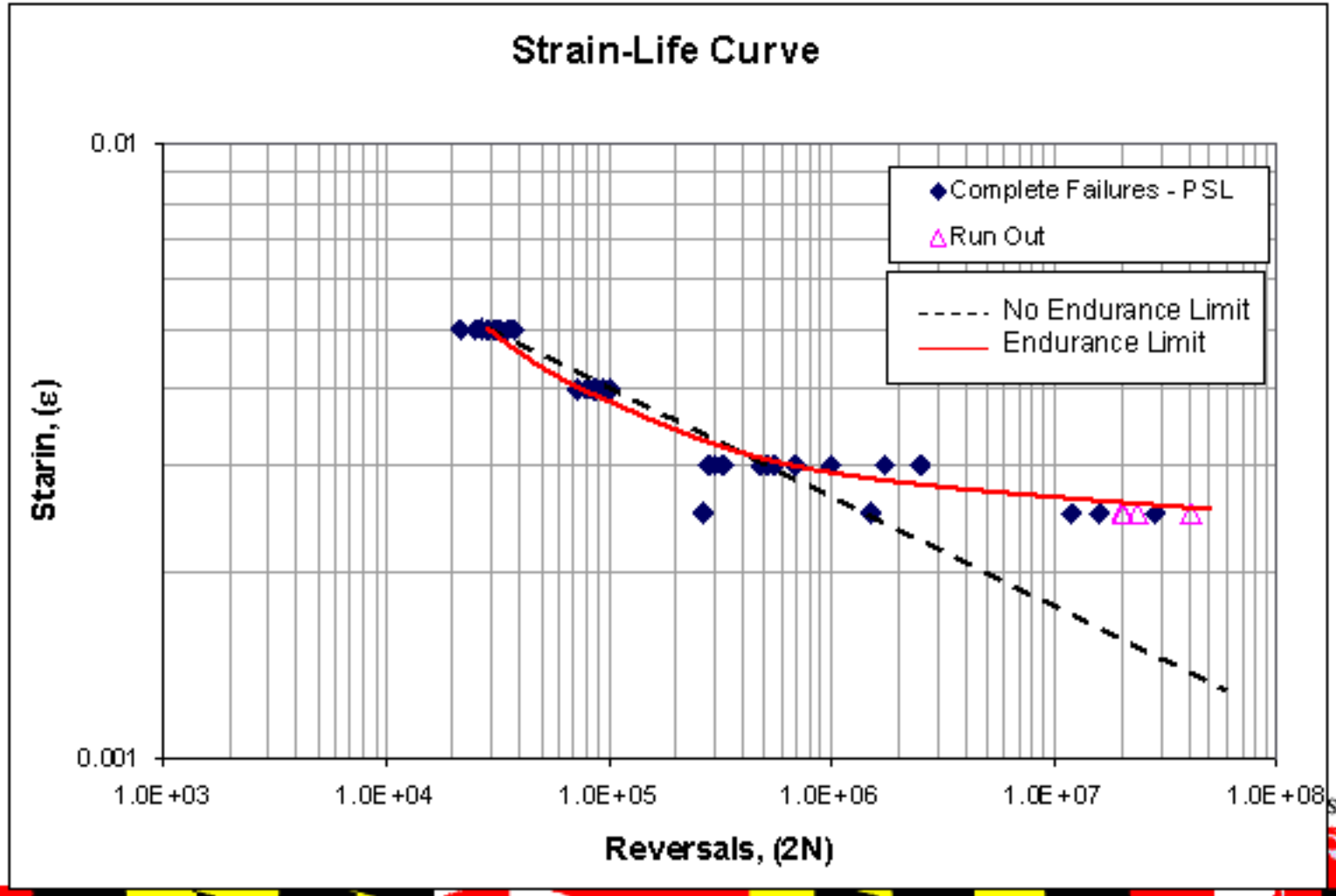
# Example 5
# A Bayesian Approach To Strain Life Model Development

# Scatter Plot of the Data



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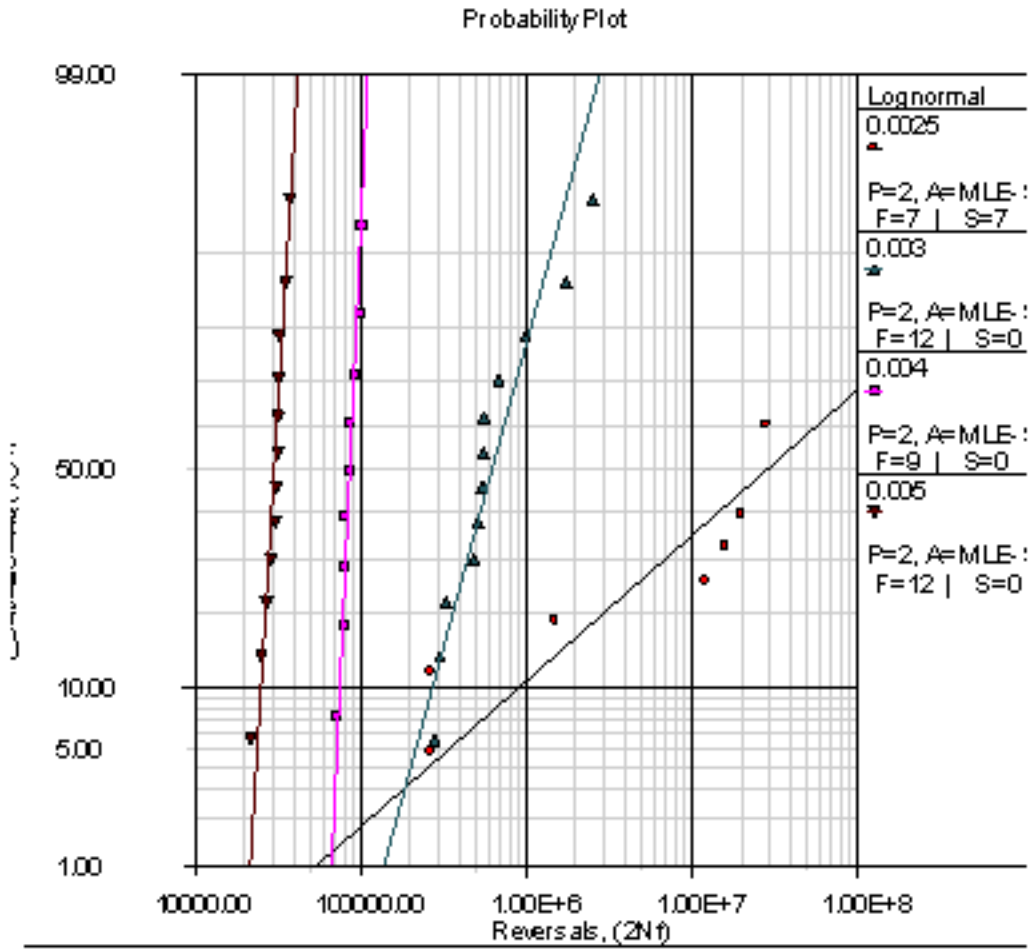
# Probability Plot of the Data




μ1=17.16, σ1=2.69
μ2=13.34, σ2=0.64
μ3=11.37, σ3=0.11
μ4=10.31, σ4=0.14



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# Observations

- The HCF experiments suggest an endurance limit for the strain
- The experimental data shows different shape parameters for life distributions at different levels of strain
- The shape parameter reduces at high strain levels



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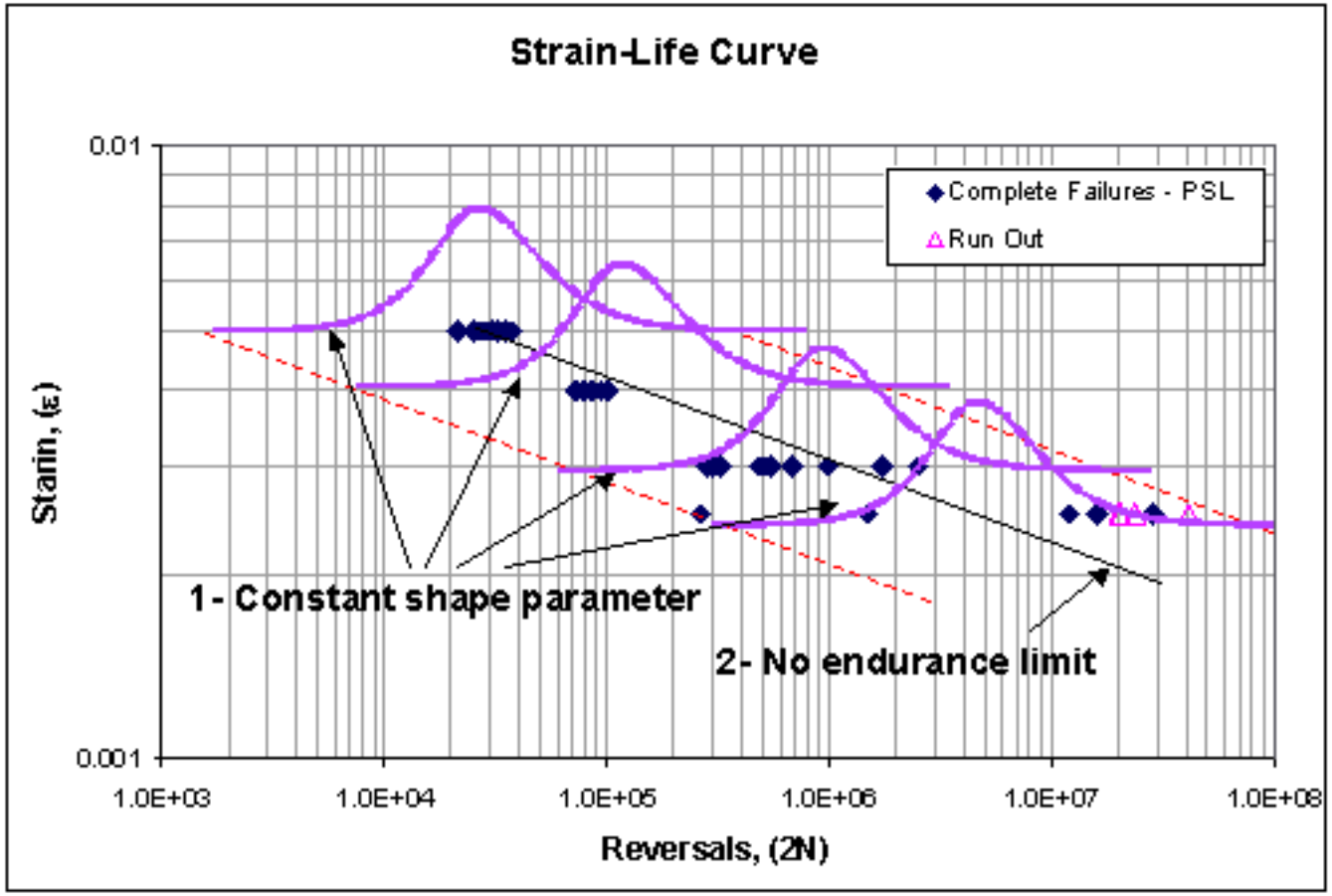
# Traditional Strain-Life Approach



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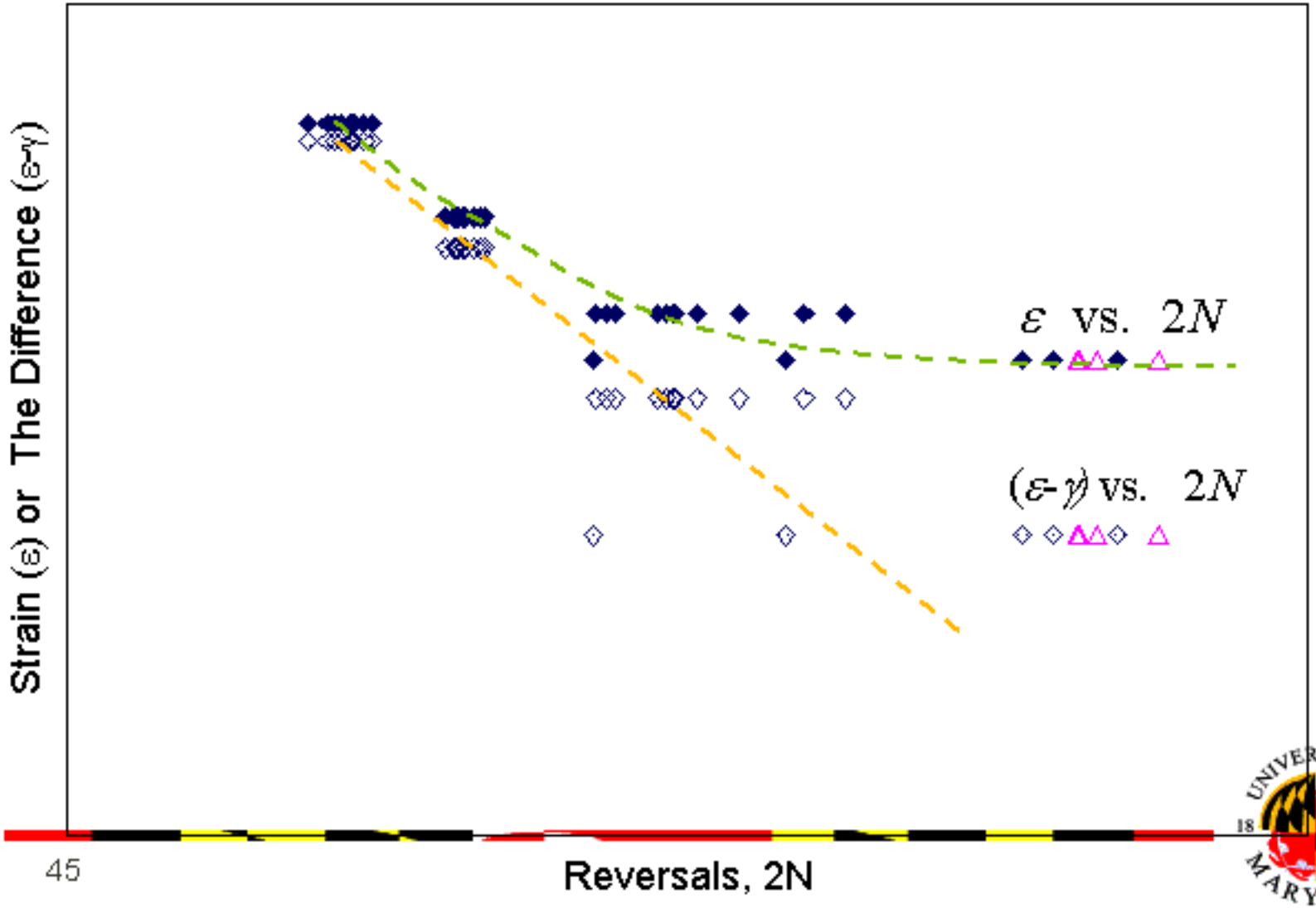
# The Notion of Endurance Limit



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# Pascual & Meeker Approach

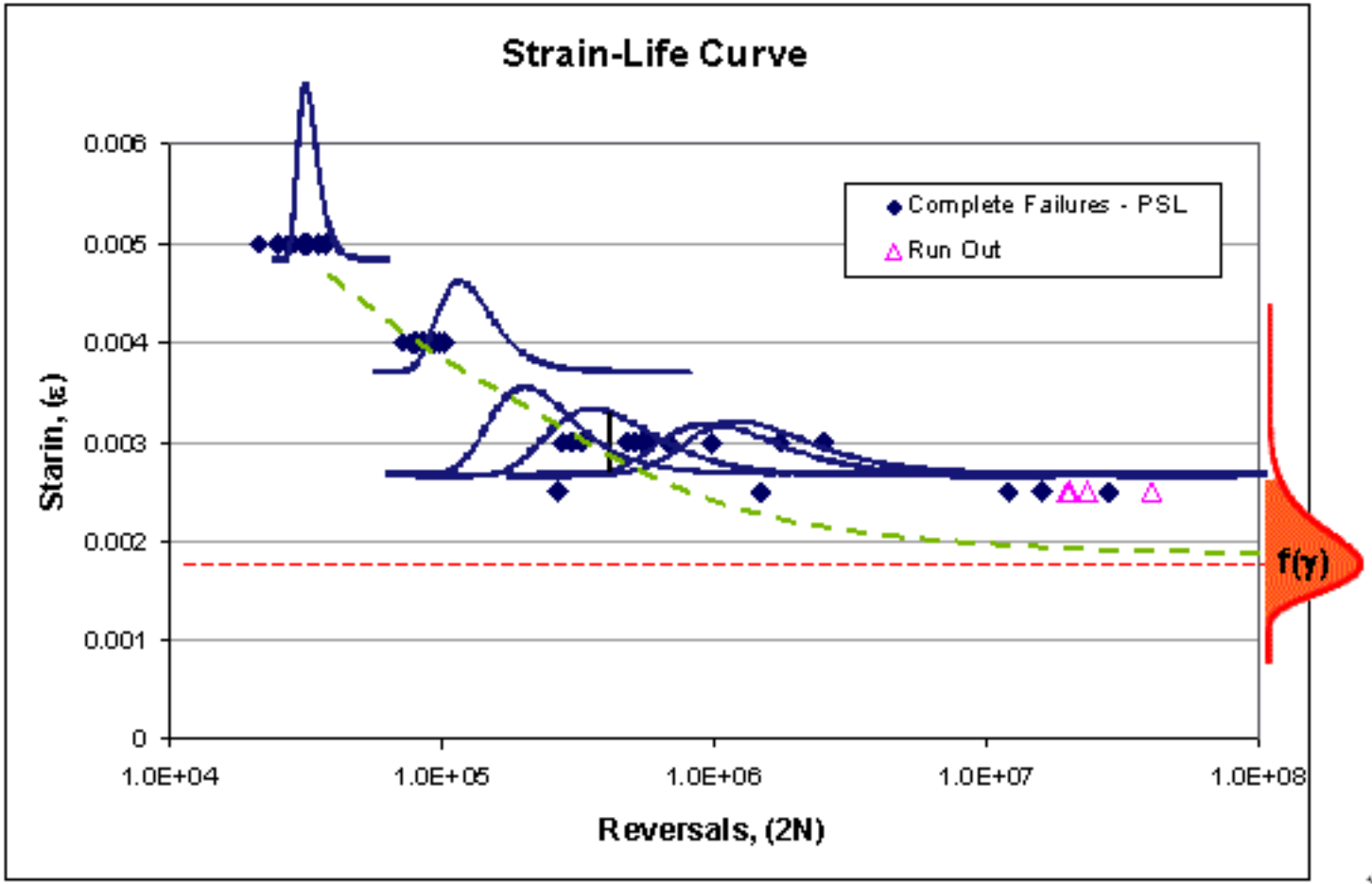
$$\log(Y) = \beta_0 + \beta_1 \log(\varepsilon - \gamma) \qquad \varepsilon > \gamma$$

$$\left.\begin{array}{l} Y = 2N \\ W = \log(Y) \\ x = \log(\varepsilon) \\ \upsilon = \log(\gamma) \\ f_\upsilon(\upsilon; \mu_\gamma, \sigma_\gamma) = \dfrac{1}{\sigma_\upsilon}\phi\left(\dfrac{\upsilon - \mu_\gamma}{\sigma_\gamma}\right) \end{array}\right\} \Rightarrow f_{W|\upsilon}(W; \beta_0, \beta_1, \sigma, x, \upsilon) = \frac{1}{\sigma}\phi\left(\frac{W - \left[\beta_0 + \beta_1 \log(e^x - e^\upsilon)\right]}{\sigma}\right)$$

$$f_W(W; x, \beta_0, \beta_1, \sigma, \mu_\upsilon, \sigma_\upsilon) = \int_{-\infty}^{x} \frac{1}{\sigma\sigma_\upsilon}\phi\left(\frac{W - \left[\beta_0 + \beta_1 \log(e^x - e^\upsilon)\right]}{\sigma}\right)\phi\left(\frac{\upsilon - \mu_\upsilon}{\sigma_\upsilon}\right)d\upsilon$$

$$F_W(W; x, \beta_0, \beta_1, \sigma, \mu_\upsilon, \sigma_\upsilon) = \int_{-\infty}^{x} \frac{1}{\sigma}\Phi\left(\frac{W - \left[\beta_0 + \beta_1 \log(e^x - e^\upsilon)\right]}{\sigma}\right)\phi\left(\frac{\upsilon - \mu_\upsilon}{\sigma_\upsilon}\right)d\upsilon$$

# Pascual & Meeker Approach (cont.)



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# Variable Shape Factor Approach

$$\log(2N) = \mu = \beta_0 + \beta_1 \log(\varepsilon - \gamma) \qquad \varepsilon > \gamma$$

$$\log(\sigma) = a + b\log(\varepsilon - \gamma) \Rightarrow \sigma = e^a (\varepsilon - \gamma)^b \qquad \varepsilon > \gamma$$

$$\left.\begin{aligned} Y &= 2N \\ W &= \log(Y) \end{aligned}\right\} \Rightarrow$$

$$f_W(W;\beta_0,\beta_1,a,b,\gamma,\varepsilon) = \frac{1}{e^a(\varepsilon-\gamma)^b}\phi\left(\frac{W-[\beta_0+\beta_1\log(\varepsilon-\gamma)]}{e^a(\varepsilon-\gamma)^b}\right)$$

$$F_W(W;x,\beta_0,\beta_1,\sigma,\mu_u,\sigma_u) = \frac{1}{e^a(\varepsilon-\gamma)^b}\Phi\left(\frac{W-[\beta_0+\beta_1\log(\varepsilon-\gamma)]}{e^a(\varepsilon-\gamma)^b}\right)$$



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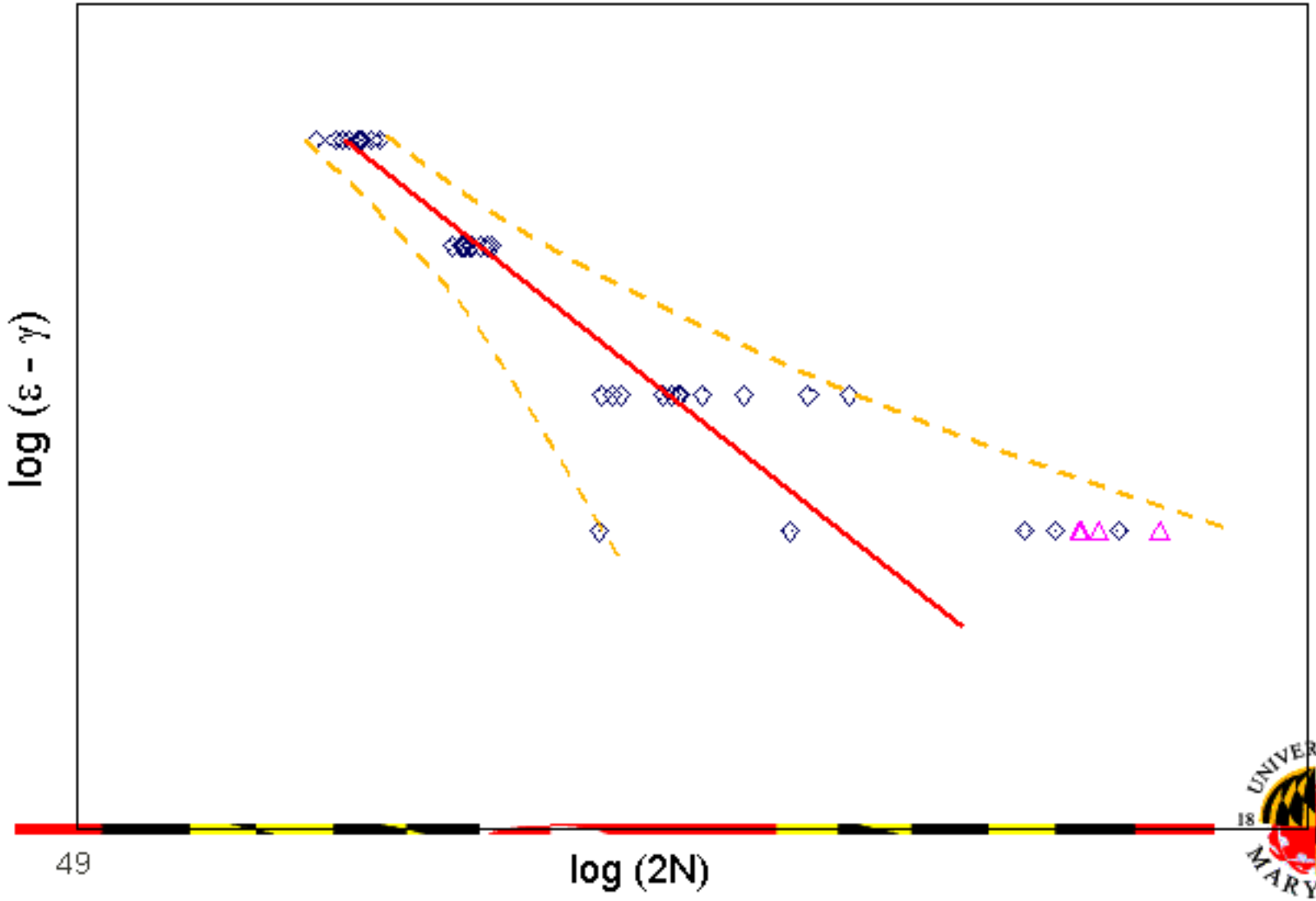
# Expected Results on (ε–γ) Scale



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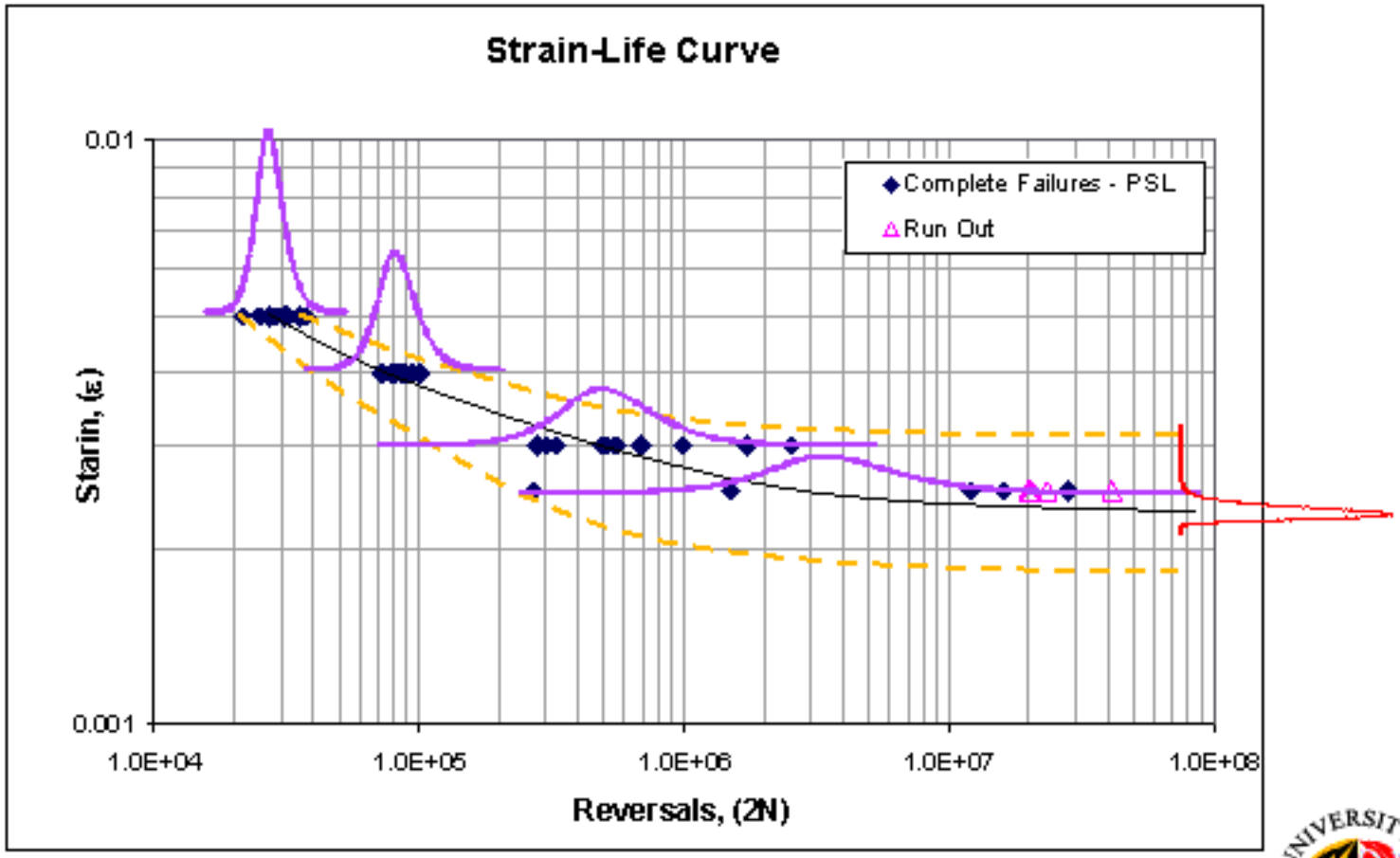
# Expected Results on (ε) Scale



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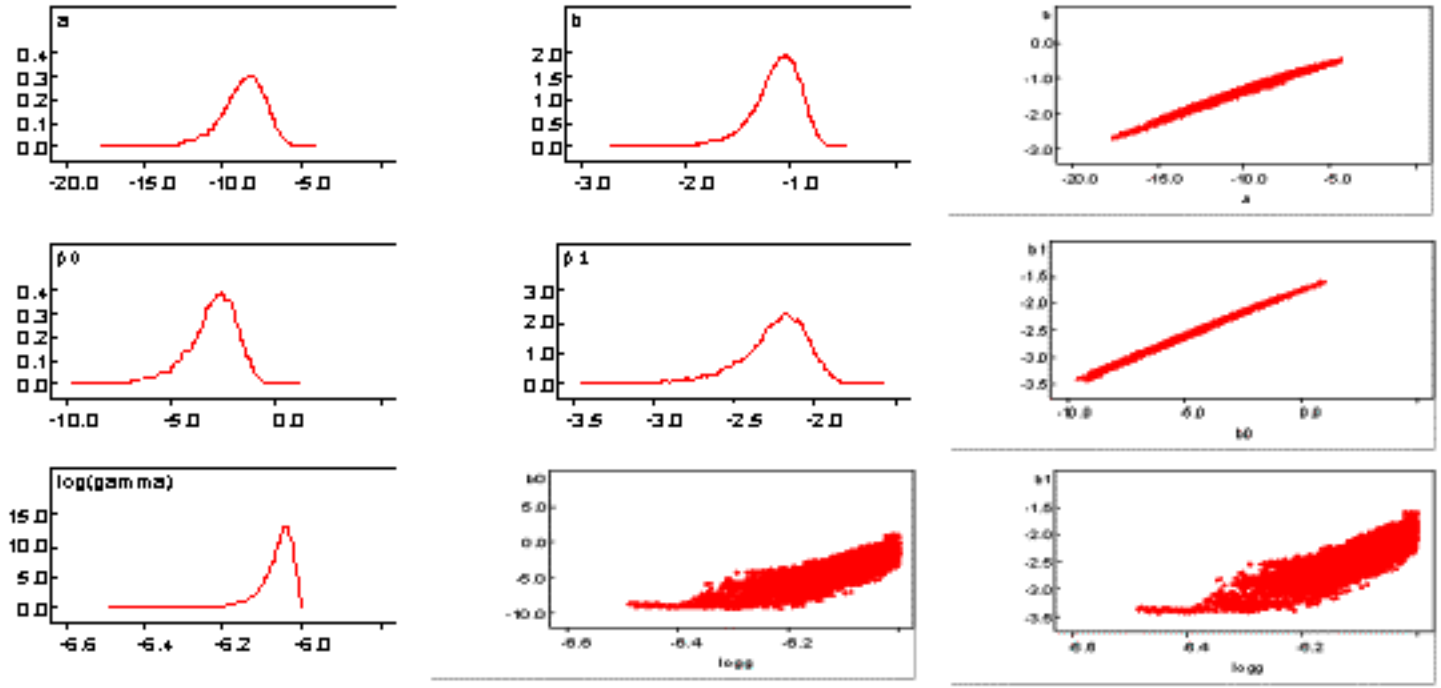
# Results




| node | mean | sd | MC error | 2.50% | median | 97.50% |
|---|---|---|---|---|---|---|
| a | -8.767 | 1.484 | 0.0499 | -12.33 | -8.587 | -6.374 |
| b | -1.139 | 0.2416 | 0.00854 | -1.742 | -1.102 | -0.7713 |
| β0 | -3.056 | 1.266 | 0.05496 | -6.188 | -2.857 | -1.133 |
| β1 | -2.258 | 0.2262 | 0.009879 | -2.827 | -2.219 | -1.92[illegible] |
| log(γ) | -6.066 | 0.0487 | 0.001991 | -6.2[illegible] | -6.054 | -6.0[illegible] |



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# Conclusions

- ✓ Uncertainty is real and instead of introducing unnecessary conservatism rely on well-established techniques and account for it.
- ✓ Save any conservatism for unknown-unknowns!



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