

# Bayesian Knowledge Fusion in Prognostics and Health Management of Structures

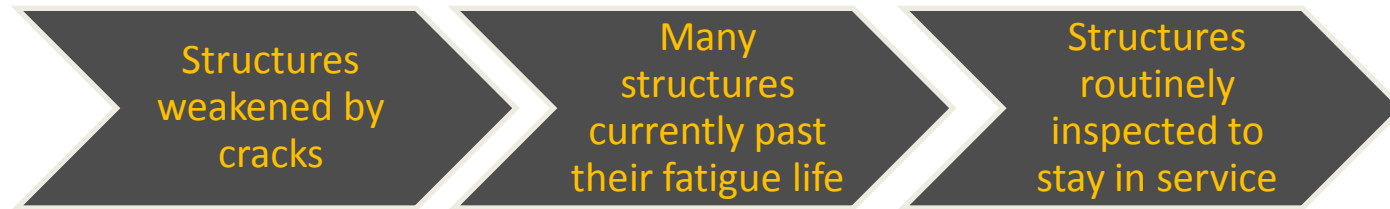
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**University of Maryland**

**Presentation**  
**At the**  
**2011 Pressure Vessels & Piping Conference**  
**July 20, 2011 Baltimore, MD**

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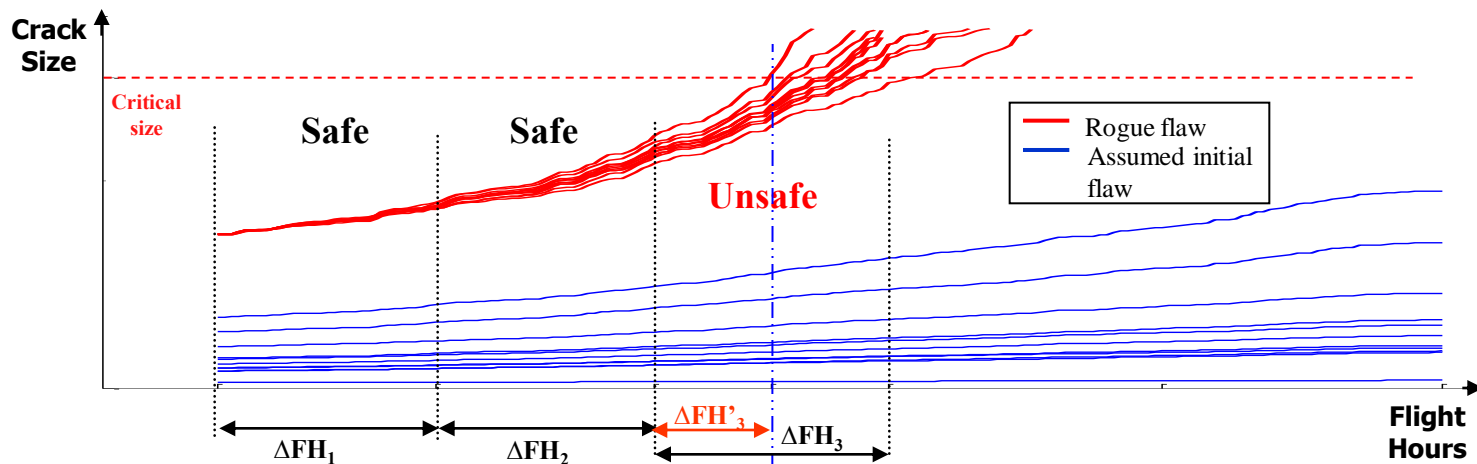
# Motivation: Structural Integrity of Aging Structures



- **Current periodic inspection** practices:
  - ✓ Labor-intensive, time consuming and expensive
  - ✓ Subject to human error
  - ✓ Inspection itself may cause damage
- **Inspection intervals** selected such that an undetected flaw will not grow to critical size before the next inspection
- **Empirical crack growth** models used to determine the inspection intervals suffer from **uncertainty**:
  - ✓ Idealized theories and simplistic assumptions may **fail to capture the underlying mechanistic failure**

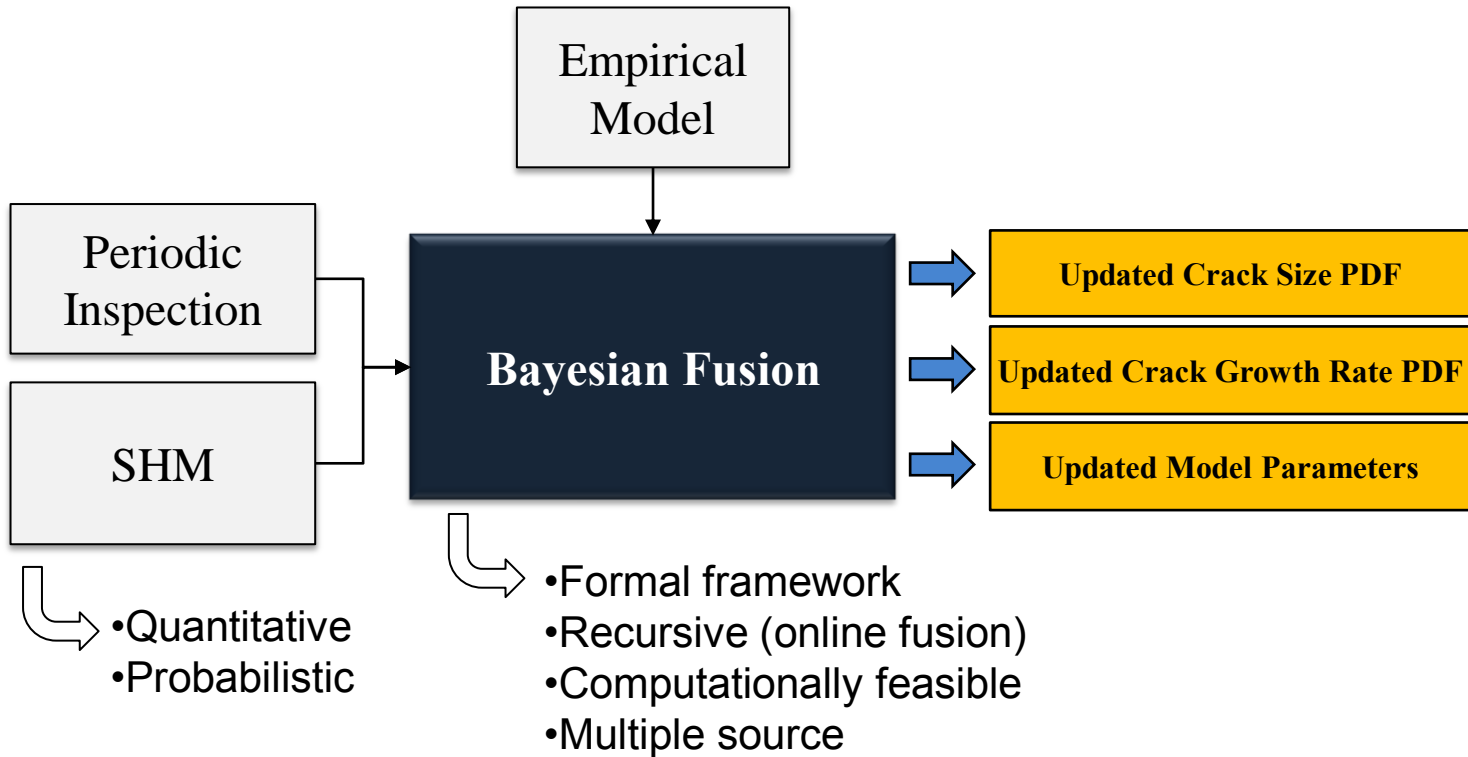
# Structural Health Management (SHM)

- **Paradigm shift: offline periodic inspections + online SHM**
- Structural health management (SHM) is the online assessment of structural integrity using appropriate NDE technology
- SHM used for:
  - ✓ **Direct assessment** of the state of structural health in real-time
  - ✓ **Provide feedback** from the structure to improve the prediction of the empirical models

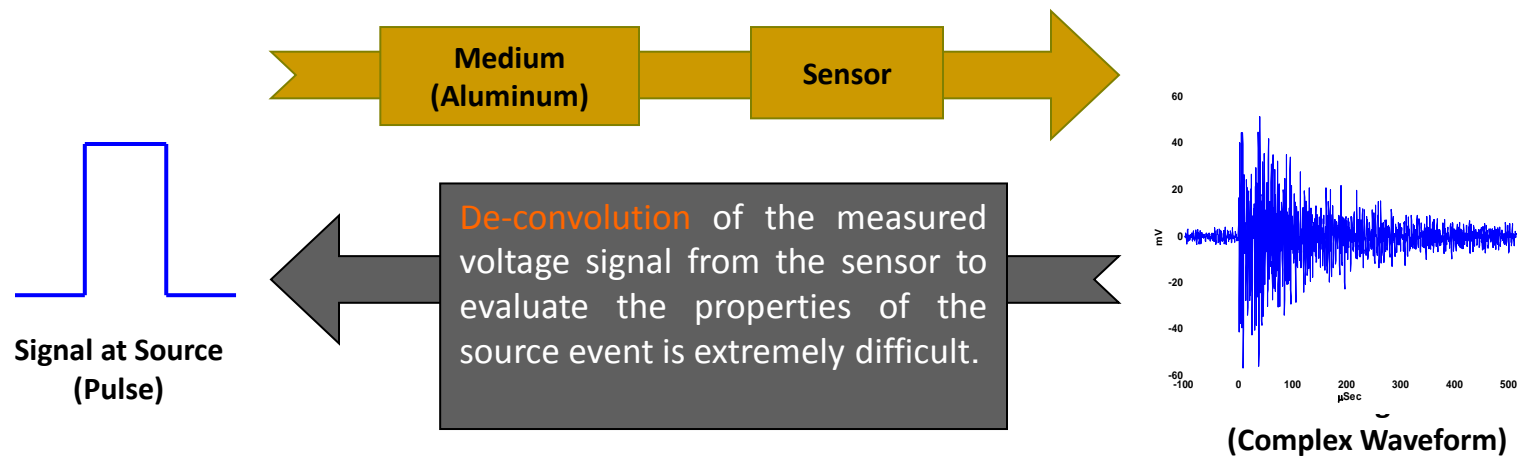
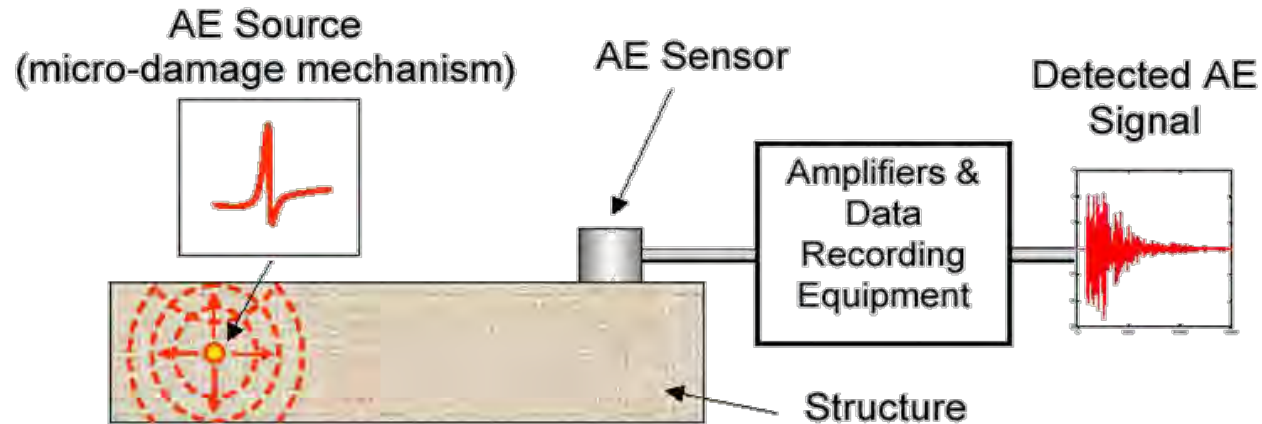


# Research Objectives and Methodology

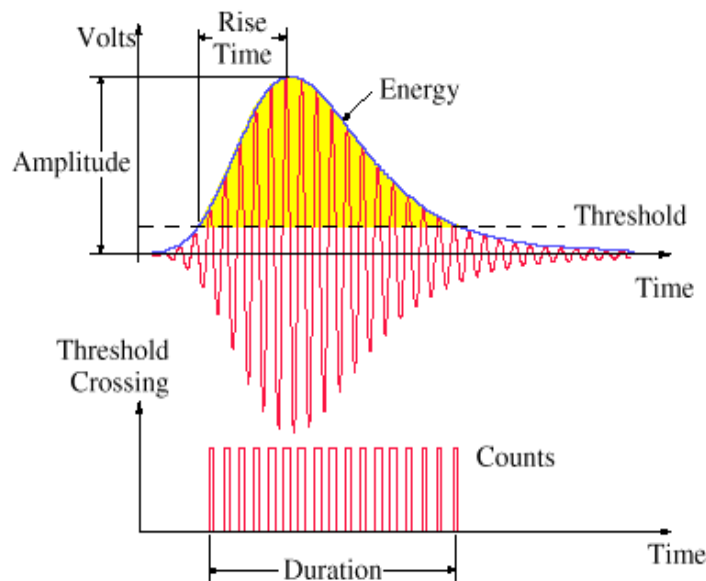
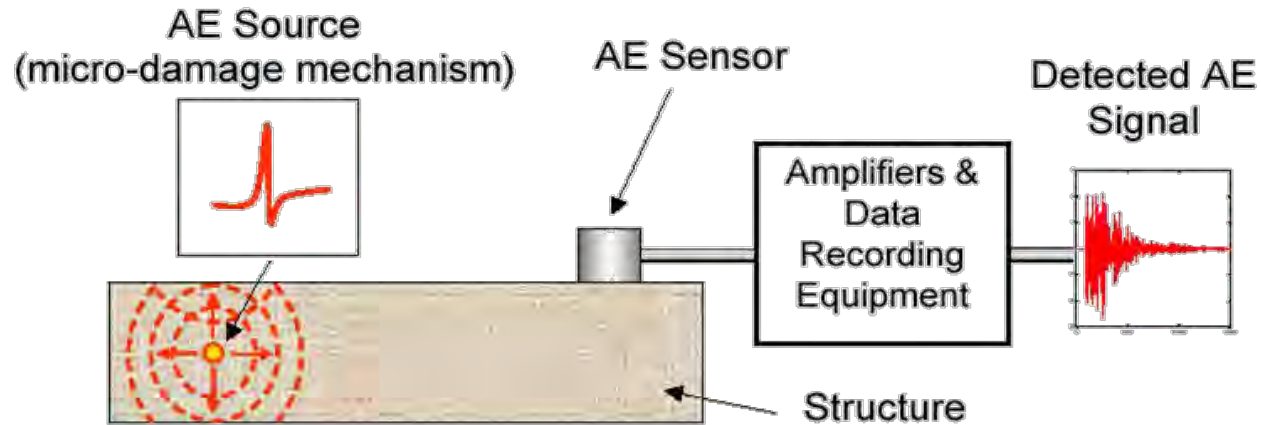
- Research Objective: Provide a hybrid framework for SHM that use sensor and visual inspection data



# AE monitoring: Theory & Background



# AE monitoring: Theory & Background (Cont.)



## AE Features

- ▶ Amplitude
- ▶ Energy
- ▶ Rise time
- ▶ **Counts (Threshold crossing)**
- ▶ Frequency content
- ▶ Waveform shape

# Correlation between AE features & Fracture parameters

## AE Features

Amplitude  
Energy  
Rise time  
Count rate ( $dc/dN$ )  
Frequency Waveform  
shape



## Fracture Parameters

$da/dN$   
 $\Delta K$

$$\frac{dc}{dN} = A_1 (\Delta K)^{A_2}$$

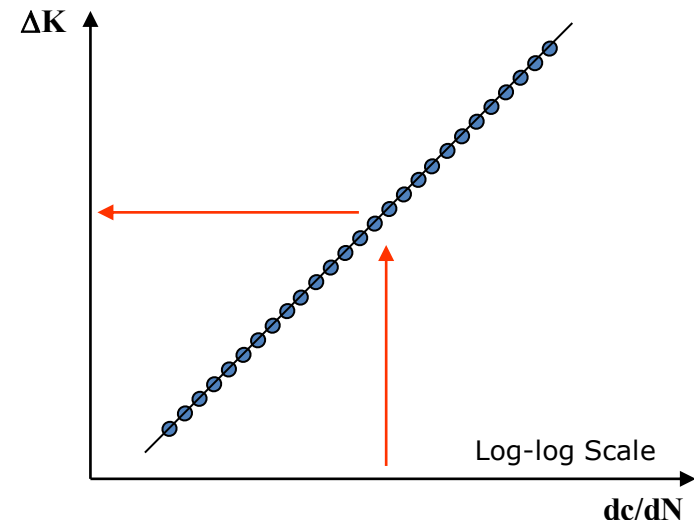


$$\Delta K = A_1^{-1/A_2} \left( \frac{dc}{dN} \right)^{1/A_2}$$



$$\log \Delta K = \alpha_1 \log \left( \frac{dc}{dN} \right) + \alpha_2$$

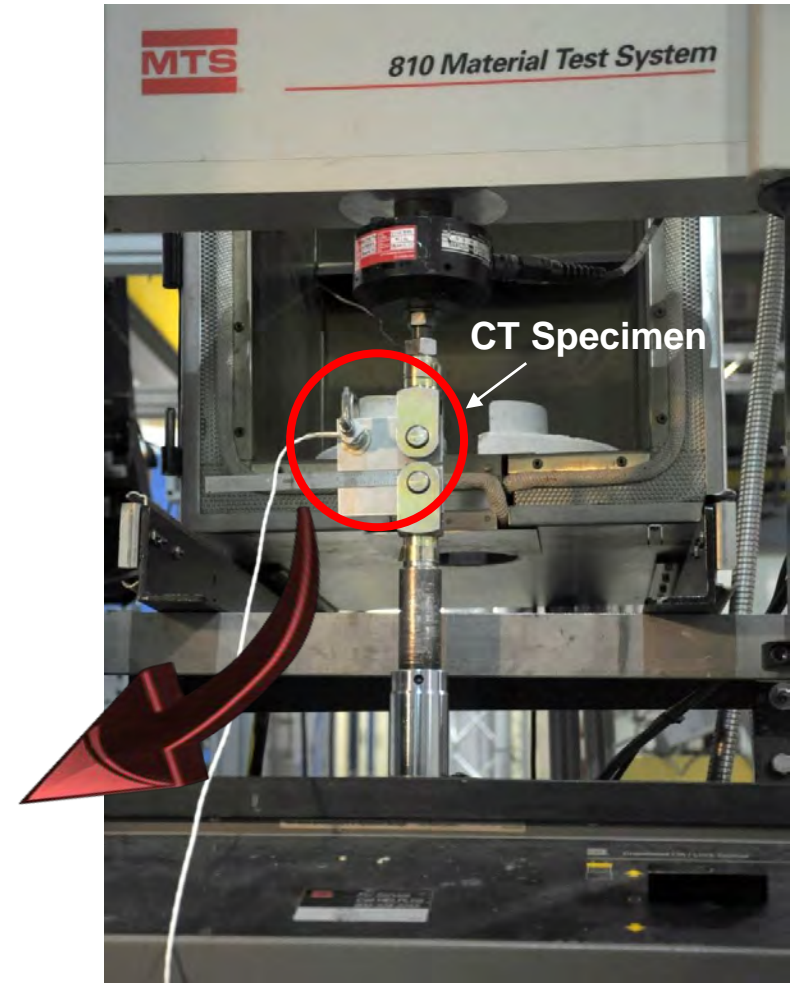
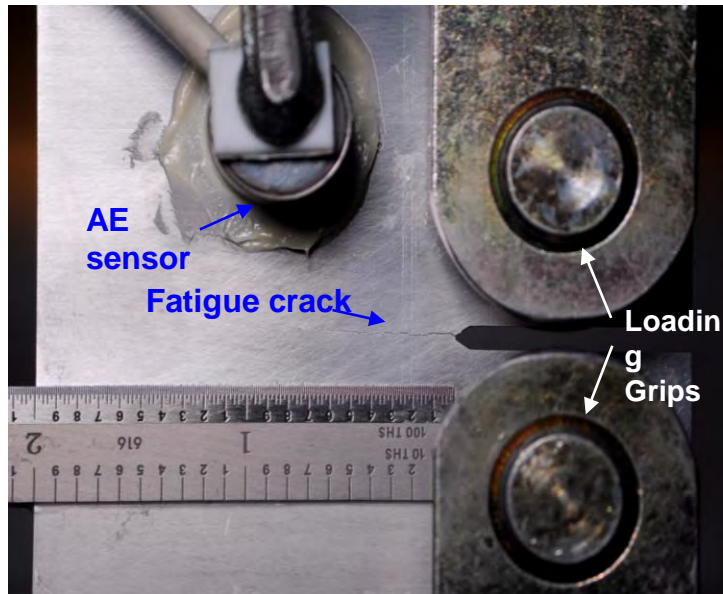
[Bassim, M.N., St Lawrence, S. & Liu, C.D., 1994. Detection of the onset of fatigue crack growth in rail steels using acoustic emission. *ENG FRACT MECH*, 47(2), 207-214.



# Crack Growth Monitoring Using AE

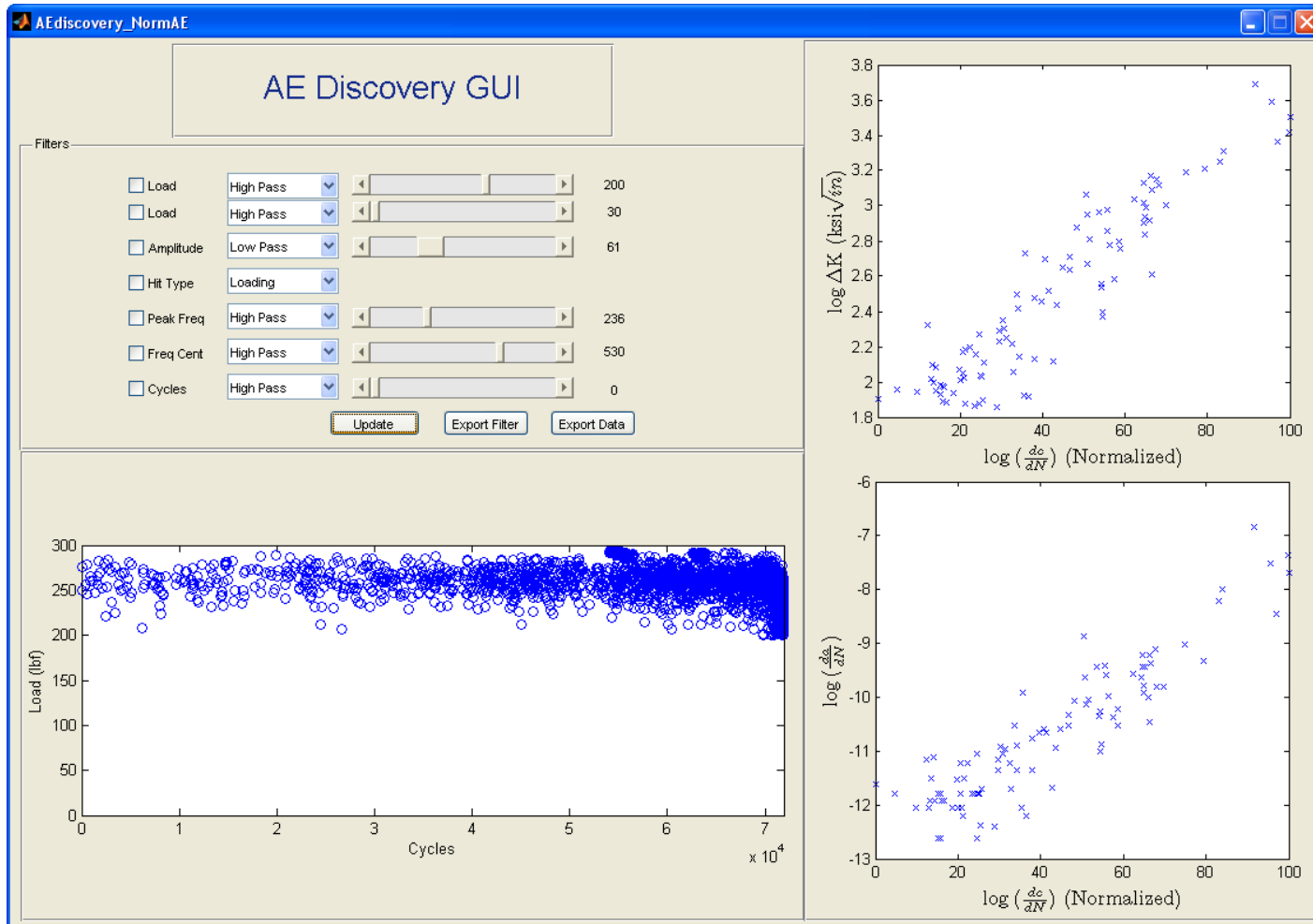
## ➤ Test objectives:

1. Investigate/verify the correlation between AE and fracture parameters.
2. Obtain experimental data needed for model development.

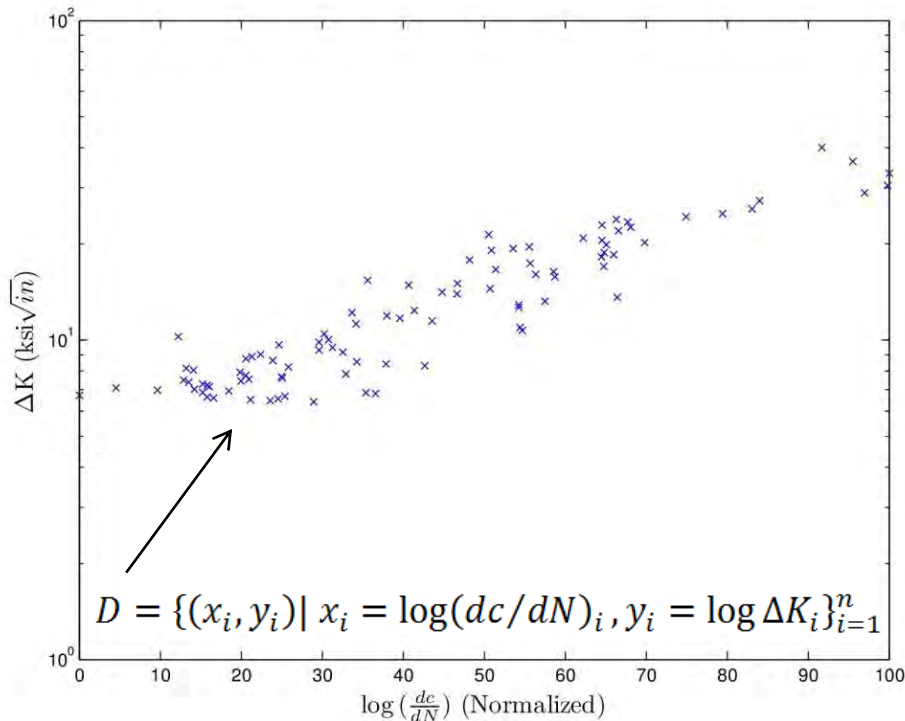




# AE Discovery MATLAB GUI



# Probabilistic Model development



$$\log \Delta K = \alpha_1 \log \left( \frac{dc}{dN} \right) + \alpha_2$$

$$y_i \sim N(\mu_i, \sigma_i)$$

$$\mu_i = \alpha_1 x_i + \alpha_2$$

$$\sigma_i = \gamma_1 \exp(\gamma_2 x_i)$$

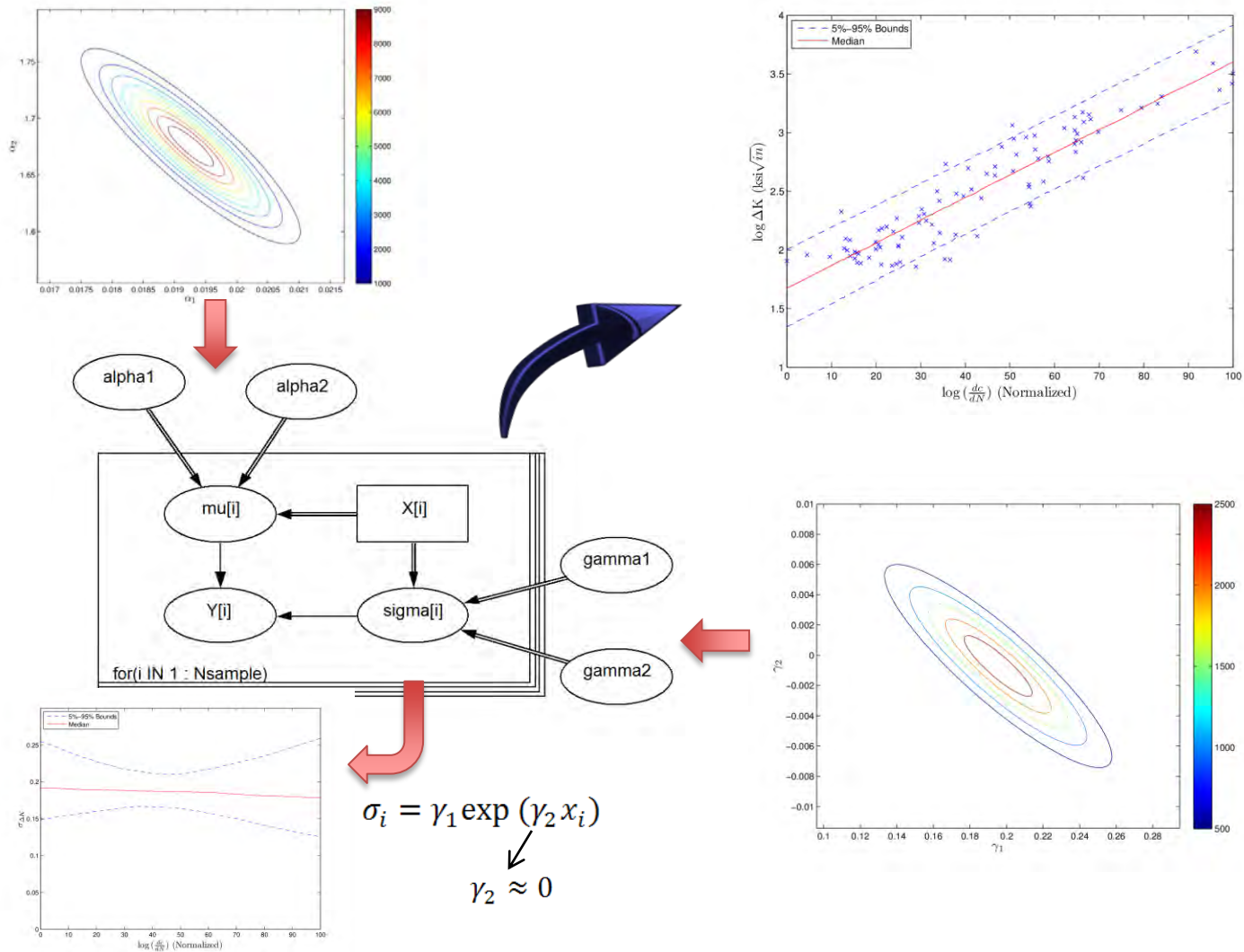
$$p(D|\alpha_1, \alpha_2, \gamma_1, \gamma_2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{y_i - (\alpha_1 x_i + \alpha_2)}{\gamma_1 \exp(\gamma_2 x_i)}\right)^2\right)$$

$$p(\Theta|D) = \frac{p(D|\Theta)p(\Theta)}{p(D)}$$

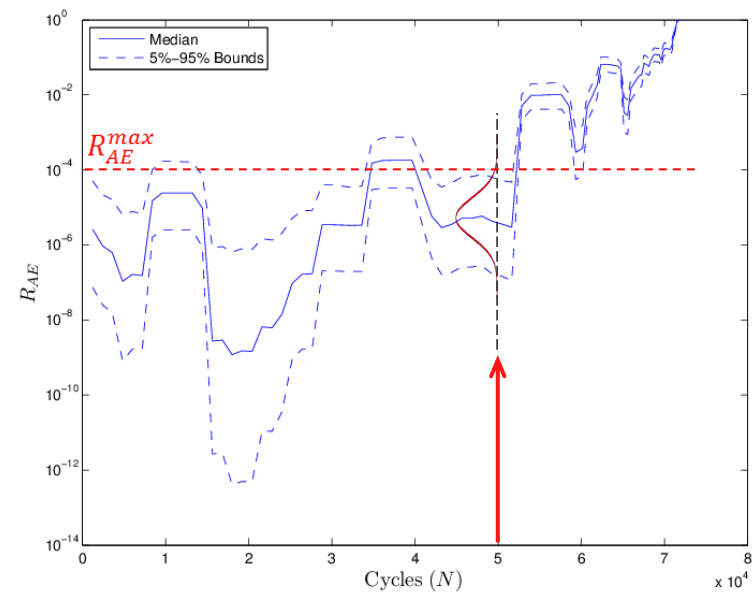
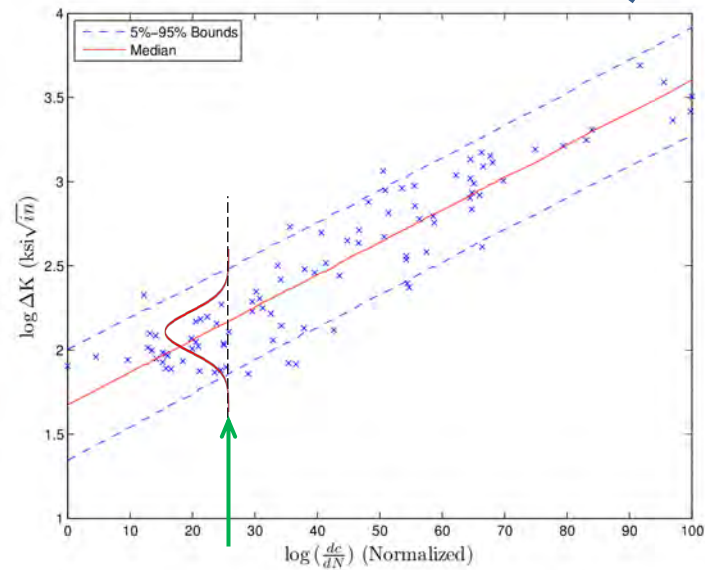
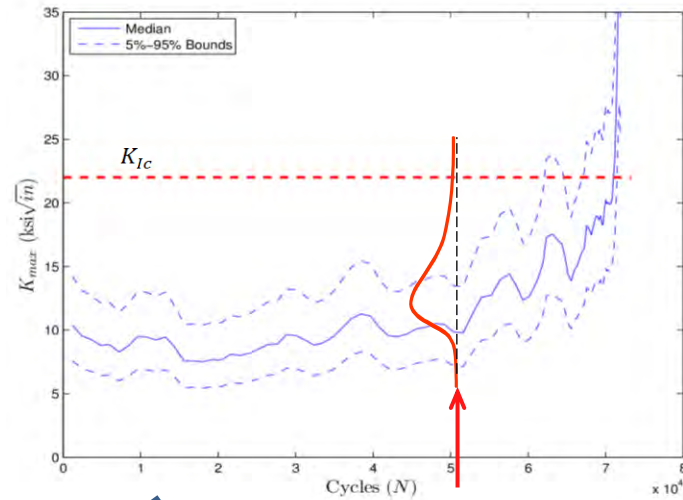
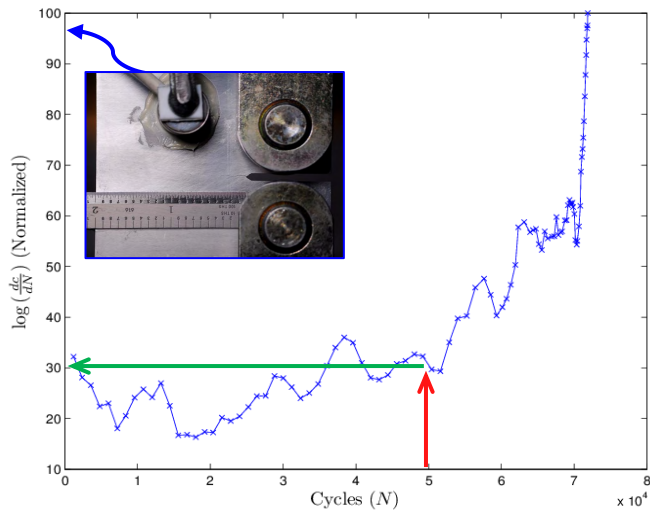
$$\Theta = \{\alpha_1, \alpha_2, \gamma_1, \gamma_2\}$$

$$p(D) = \int p(D|\Theta)p(\Theta)d\Theta$$

# Bayesian Regression

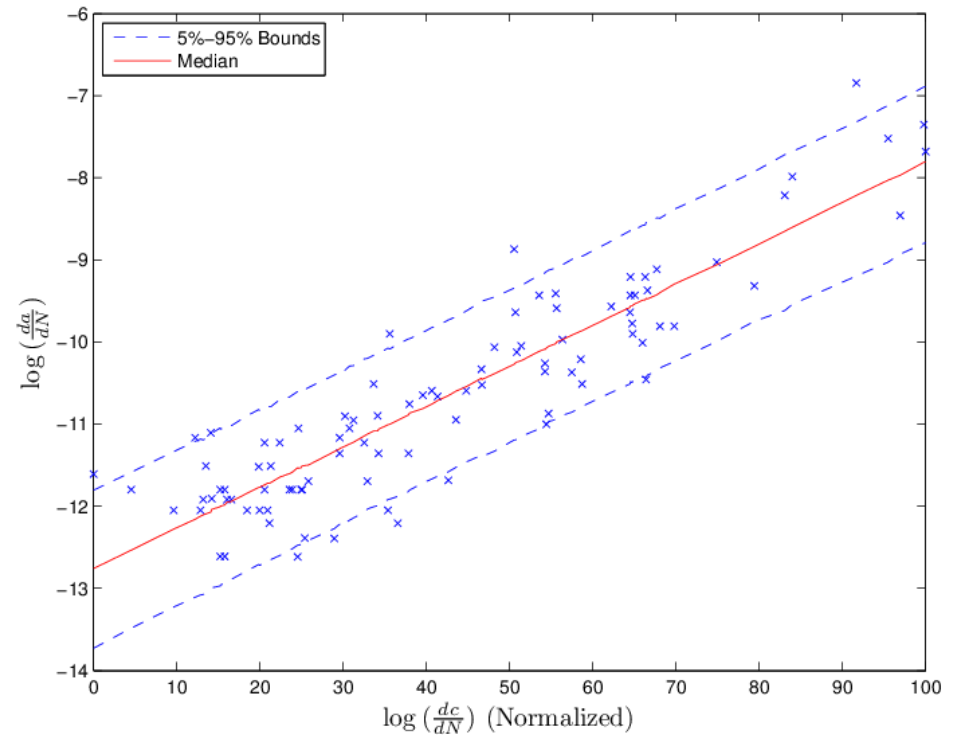
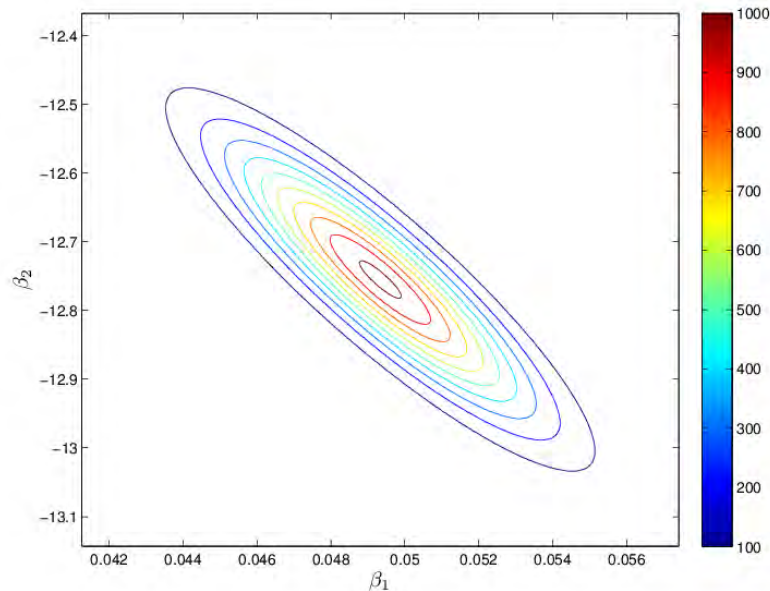
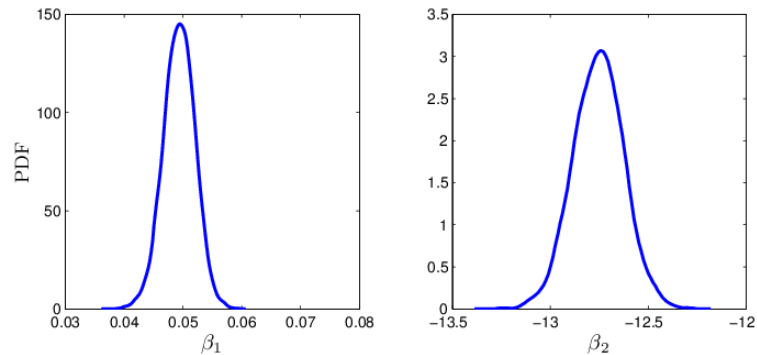


# AE-Based Risk Factor

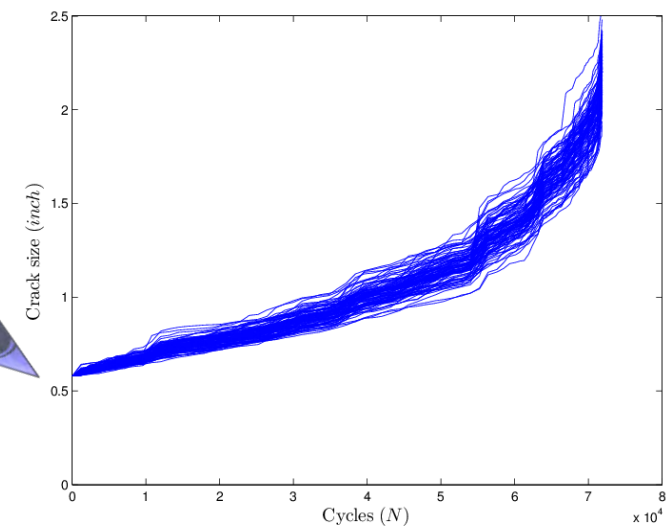
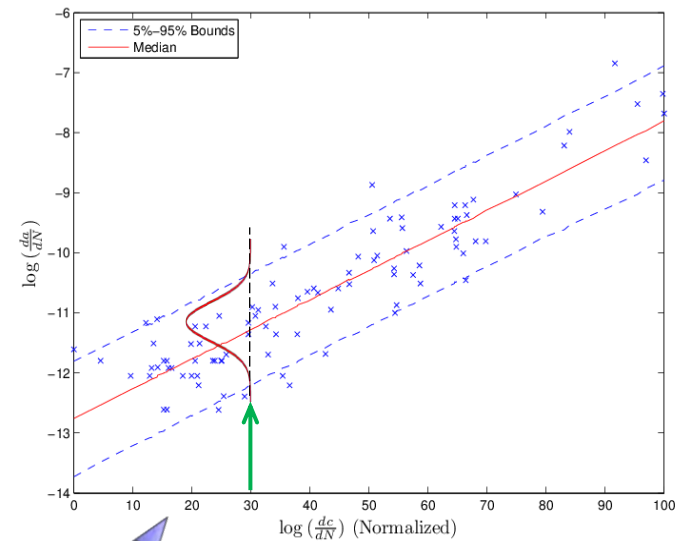
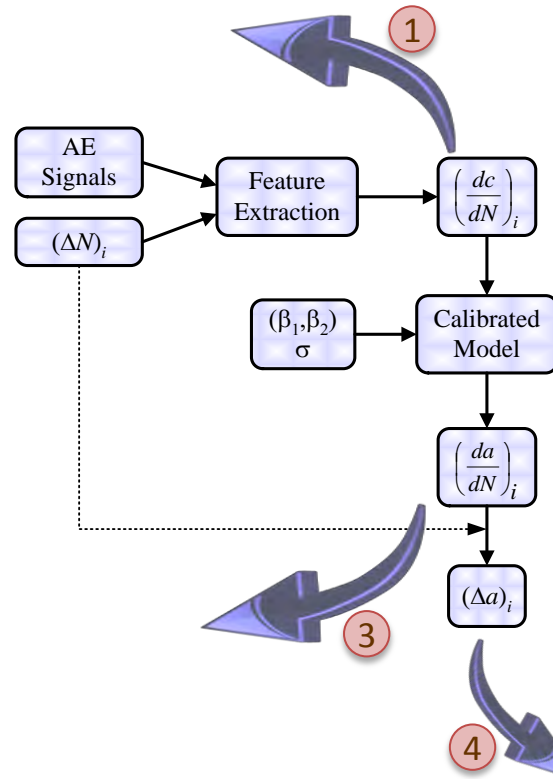
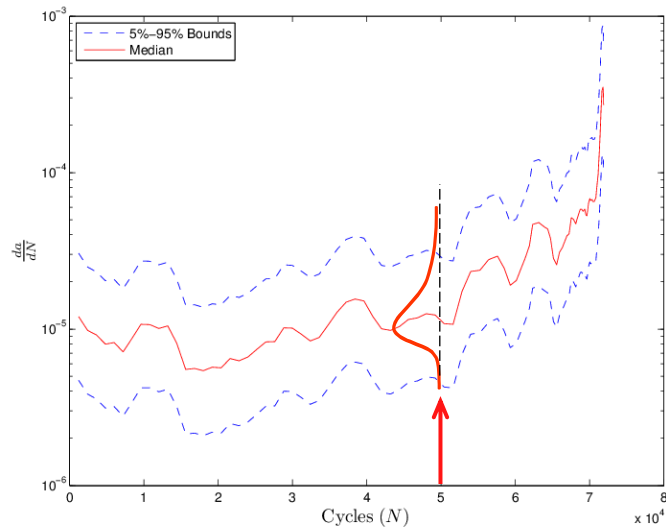
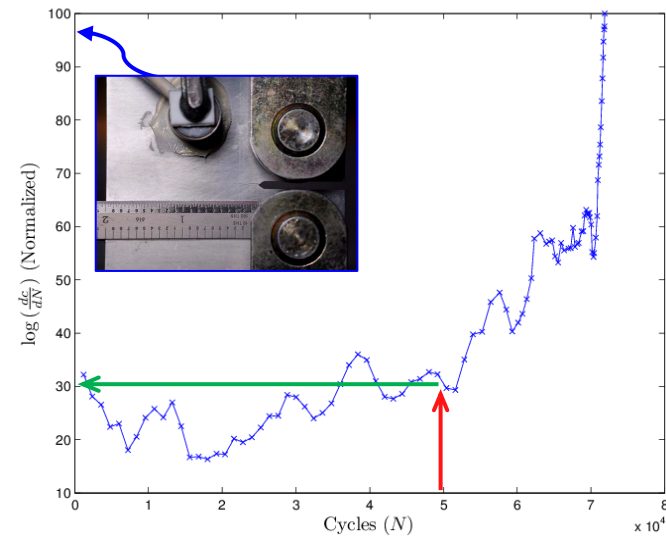


# AE-based Crack Size Estimation

$$\log\left(\frac{da}{dN}\right) = \beta_1 \log\left(\frac{dc}{dN}\right) + \beta_2$$

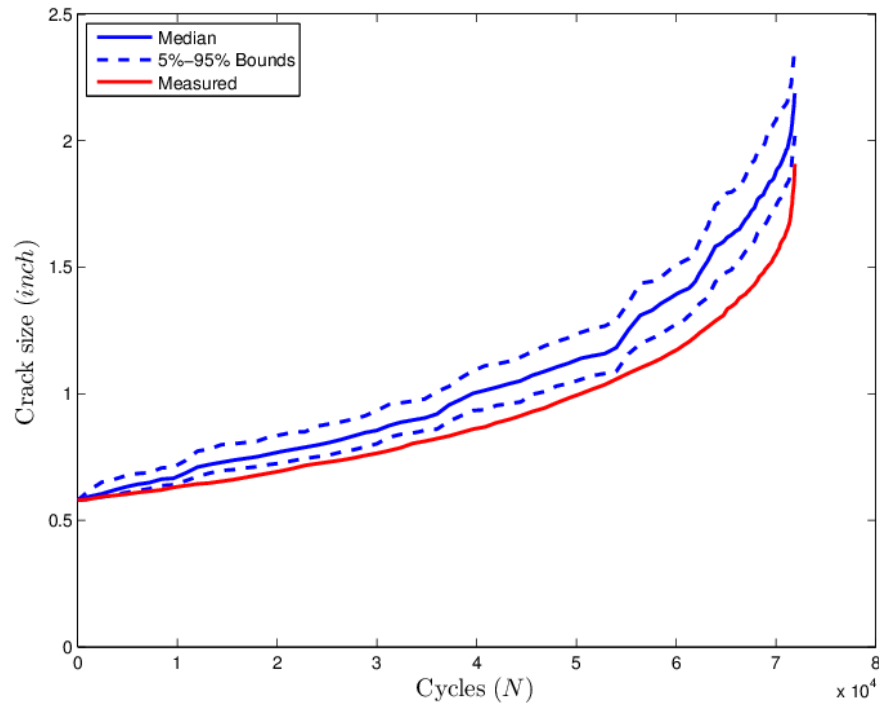


# AE-Based Crack Size Estimation

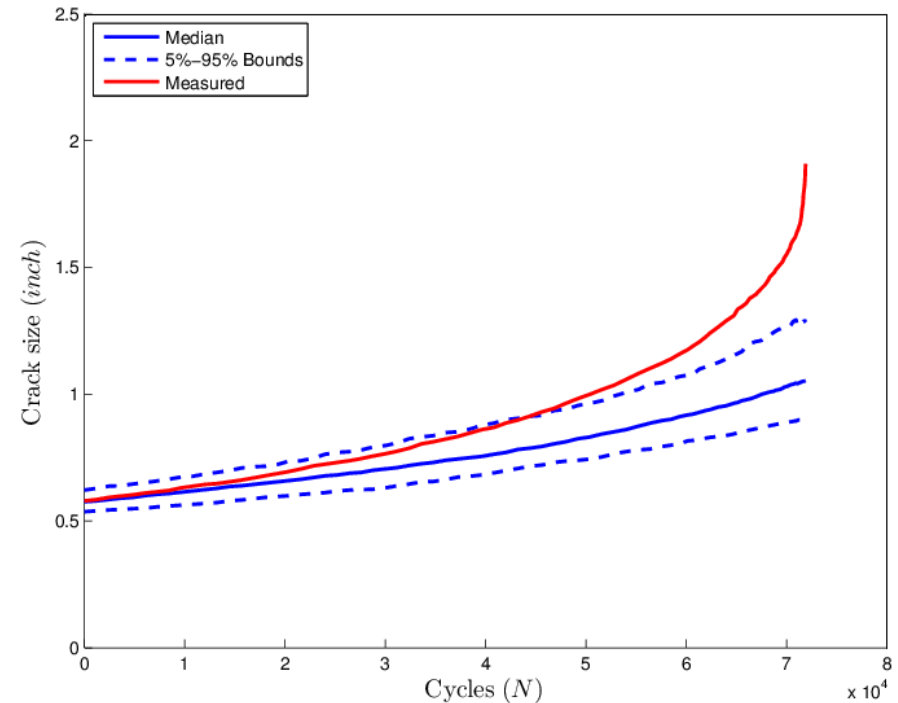


# AE-Based vs. Empirical Crack Growth Model

## AE-Based Crack Growth Model

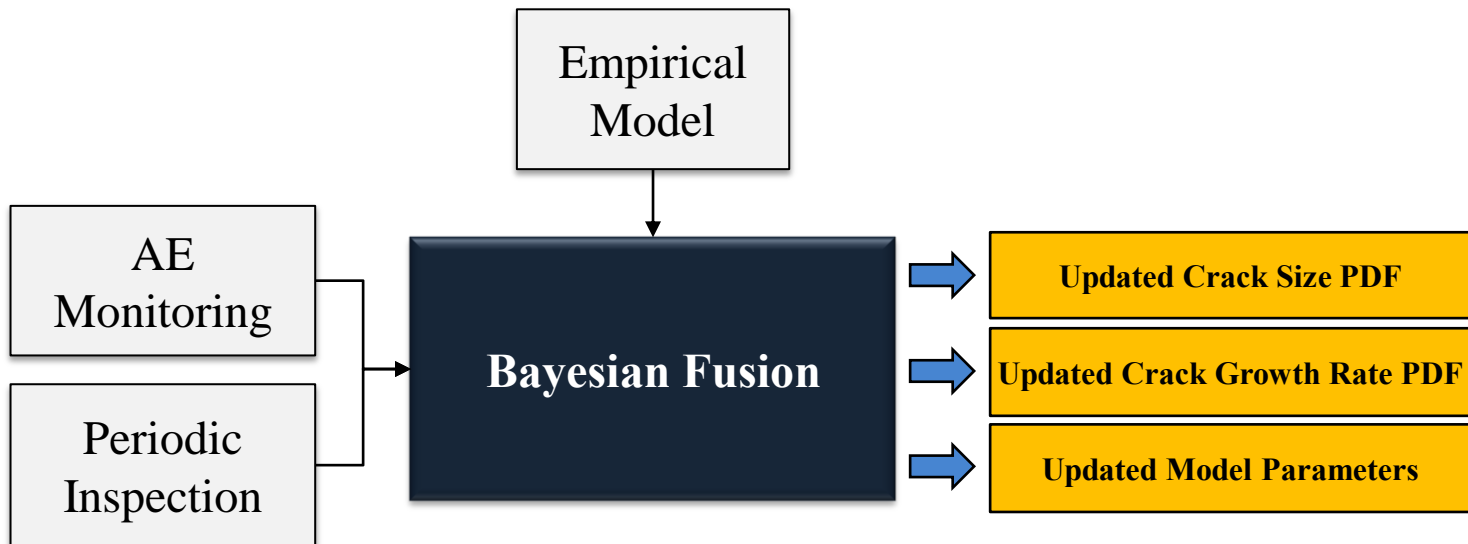


## Empirical Crack Growth Model



# Bayesian Fusion

- The necessary information for developing a structural health diagnostic and prognostic (i.e., SHM) solution is often obtained from various sources.





# Dynamic State-Space Model

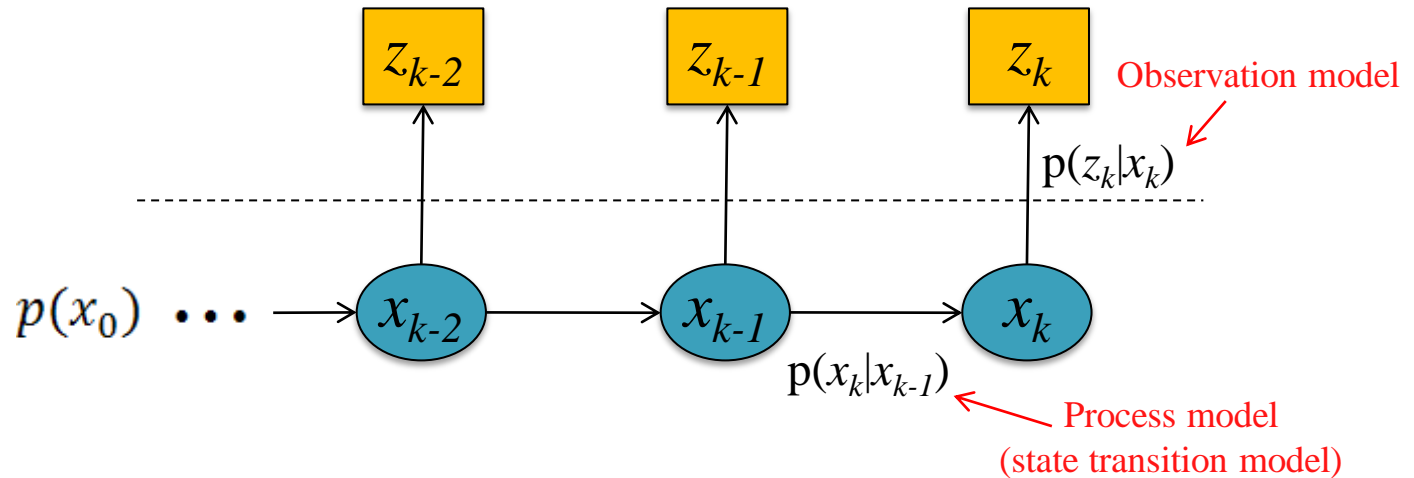
- Recursive Bayesian estimation* is a probabilistic approach for estimating an unknown probability density function **recursively over time** using incoming uncertain observation (noisy measurements) and a mathematical process model that describes the evolution of the state variables over time.

Observations  
at time step  $k$ :

$$z_k \in \mathbb{R}^m$$

State variable  
at time step  $k$ :

$$x_k \in \mathbb{R}^n$$



Key assumptions:

- States follow a first order Markov process.
- Observations independent given the states.

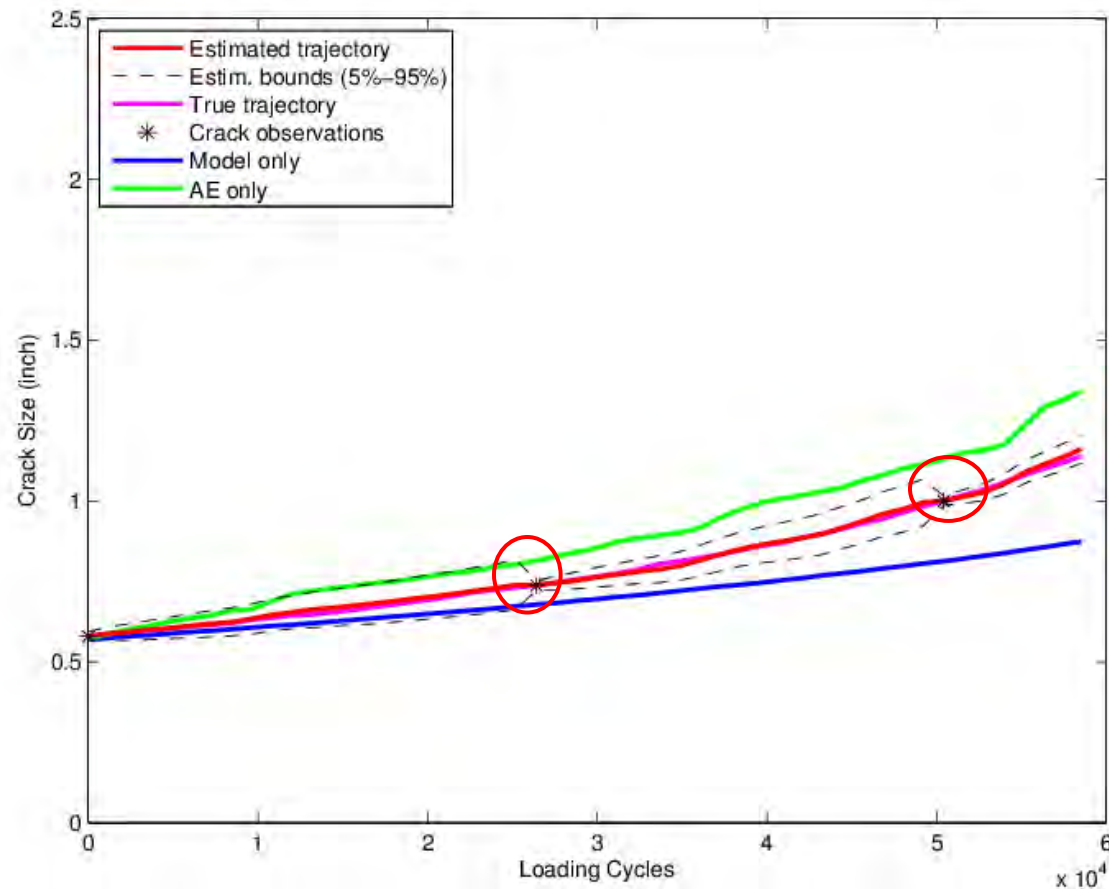
We are interested in posterior distribution of state  $x_k$ , given the time series of past observations:

$$p(x_k | x_{k-1}, x_{k-2}, \dots, x_1) = p(x_k | x_{k-1})$$

$$p(z_k | x_k, z_{k-1}, \dots, z_1) = p(z_k | x_k)$$

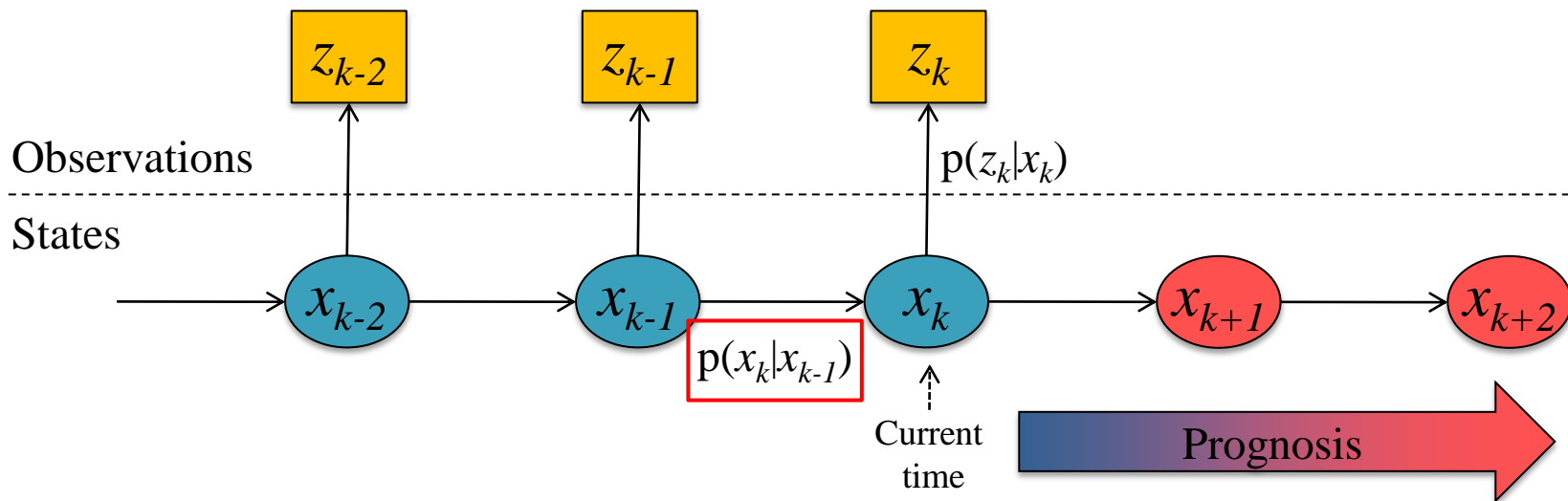
$$p(x_k | z_k, z_{k-1}, \dots, z_1) = ?$$

# Results: Effect of Frequency of Inspections



# Prognosis: Predicting Future Crack Size

- Step1: Update the empirical model parameters (process model) given the crack size and rate observations.
- Step2: Predict future crack size given assume usage profile.
  - ✓ Estimate remaining useful life (RUL)
  - ✓ Estimate probability of exceeding critical crack size



# Prognosis Results

