Bayesian Knowledge Fusion in Prognostics and Health Management—A Case Study

Masoud Rabiei
Mohammad Modarres
Center for Risk and Reliability
University of Maryland-College Park

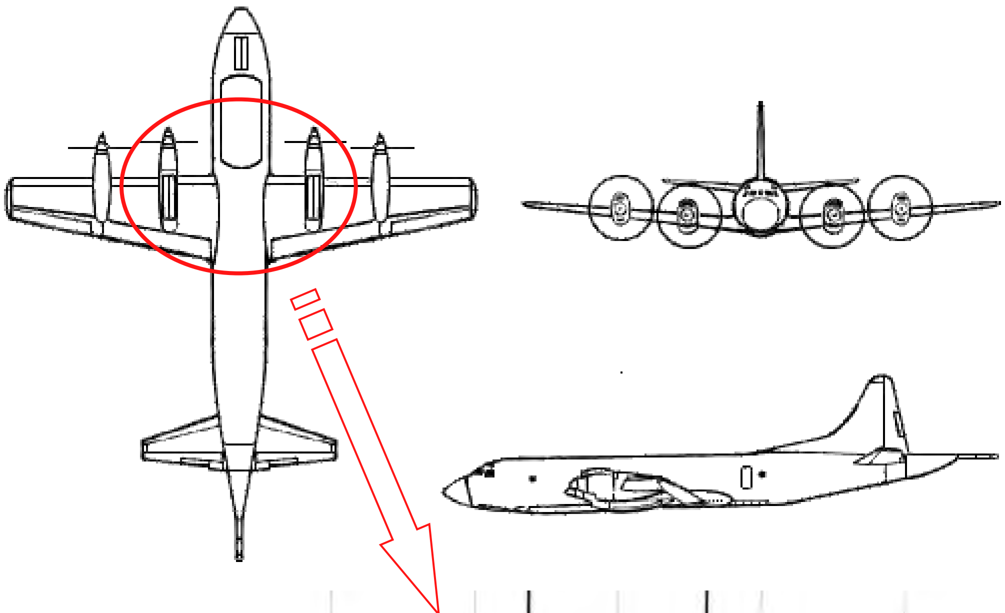
Ali Moahmamd-Djafari
Laboratoire des Signaux et Systèmes, UMR
8506 CNRS - SUPELEC - Univ Paris Sud 11

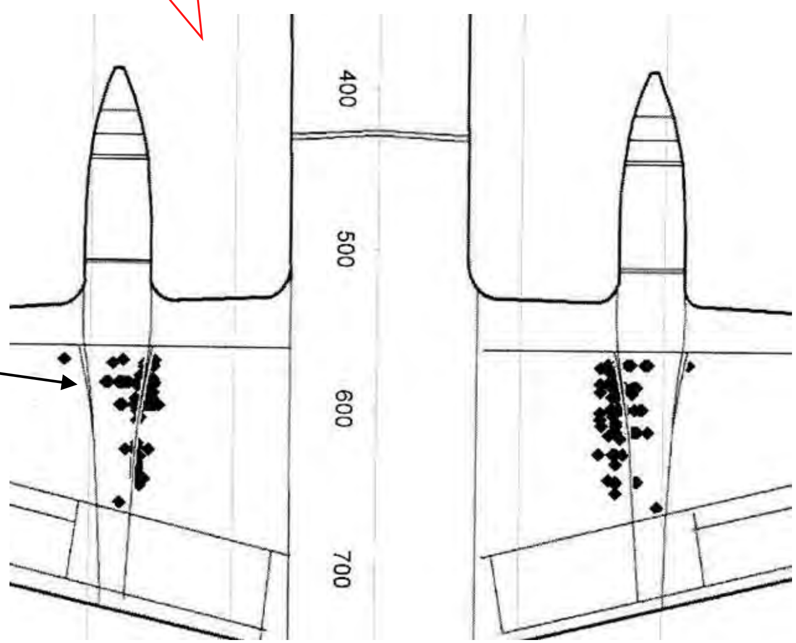
A. JAMES CLARK
SCHOOL OF ENGINEERING

Presentation at the MaxEnt2010
July 2010
Chamonix, France

Motivation and Background

P3 Aircraft Health Management

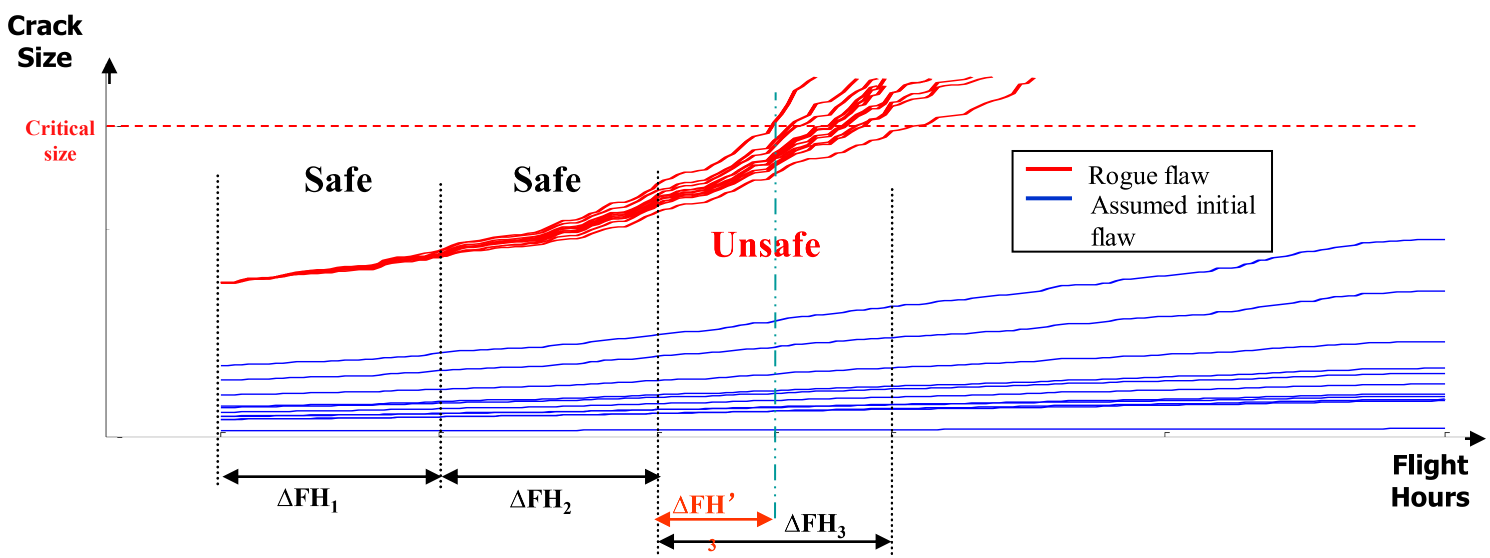




Fatigue Cracks

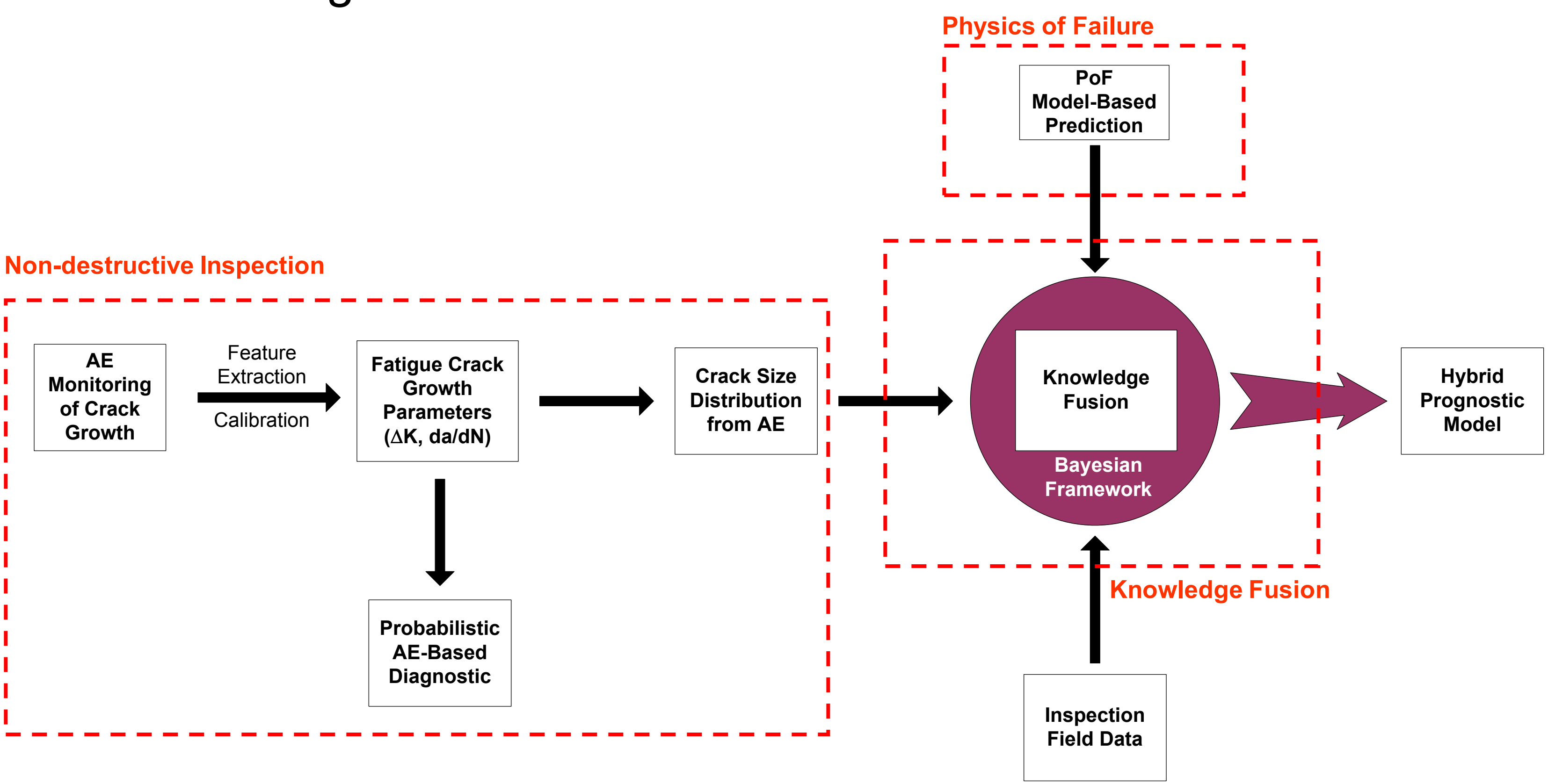
- SAFE-life design assumes very low probability of crack initiation
- Full-sacle fatigue tests with 2X safety factors
- Objective: Quantify the risk associated with fleet life extension (damage-tolerance regime)

- Today's objective in fleet management is to use an airframe to its maximum service life (*total life*) [1]
- Stochastic Physics-of-Failure (PoF) approach has proved useful for fleet management.
- Shortcomings of PoF:
  1. Limited knowledge about the underlying physics of failure
  2. Scarcity of relevant material-level test data to estimate model parameters
  3. In practice, disconnected from the system being modeled (no feedback)

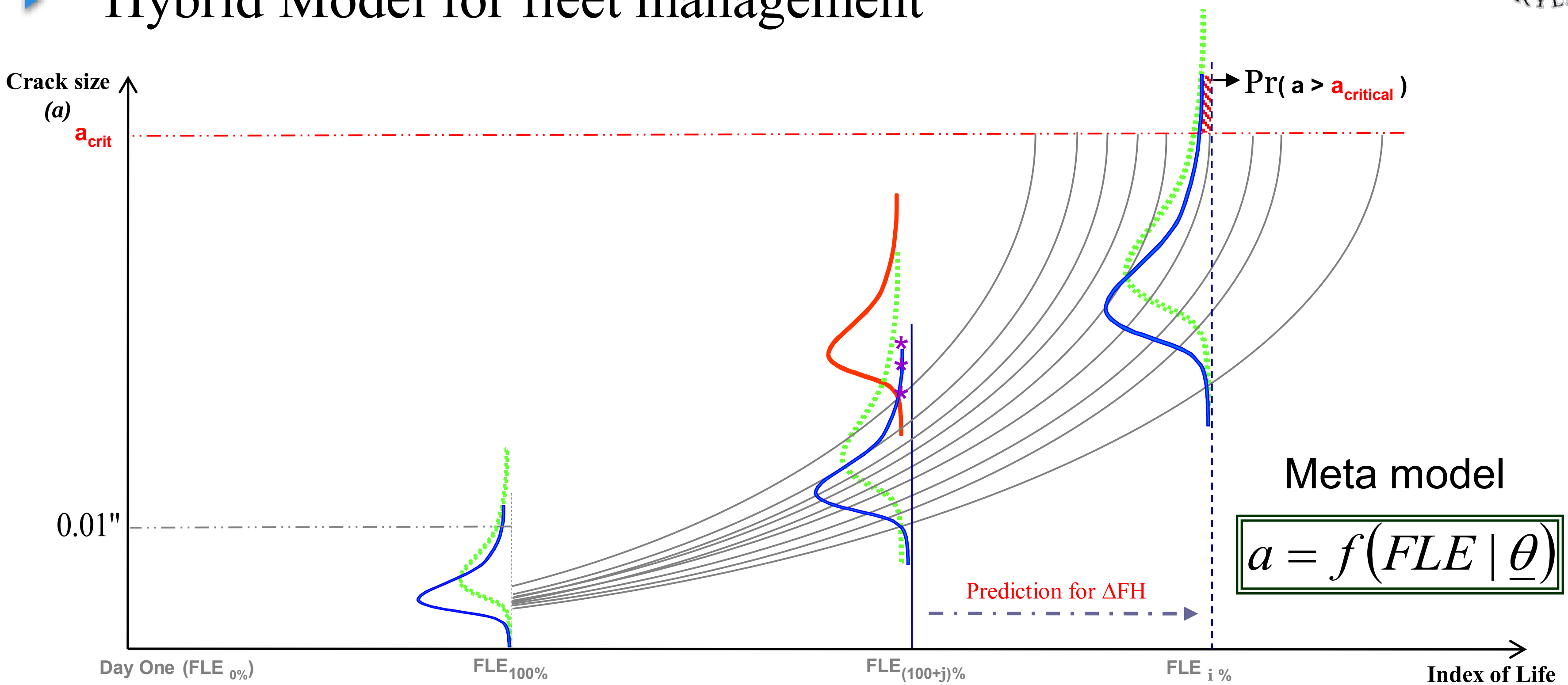


[1] Hoffman, Paul C. Fleet management issues and technology needs. *International Journal of Fatigue* 31, no. 11-12 (November): 1631-1637.

- GOAL: Developing a hybrid prognostics methodology for health management consisting of the following modules:
  - Physics-of-Failure (PoF) Model
  - NDI-based structural integrity assessment
  - Knowledge Fusion Module



- Hybrid Model for fleet management



Crack Growth PoF Model

On-board NDI

Field Inspection

$$\pi\left(\underline{\theta}\mid \text{Evidence}\right)=\frac{\mathrm{L}\left(\text{Evidence}\mid \underline{\theta}\right)p\left(\underline{\theta}\right)}{p\left(\text{Evidence}\right)}$$

Acoustic Emission Monitoring

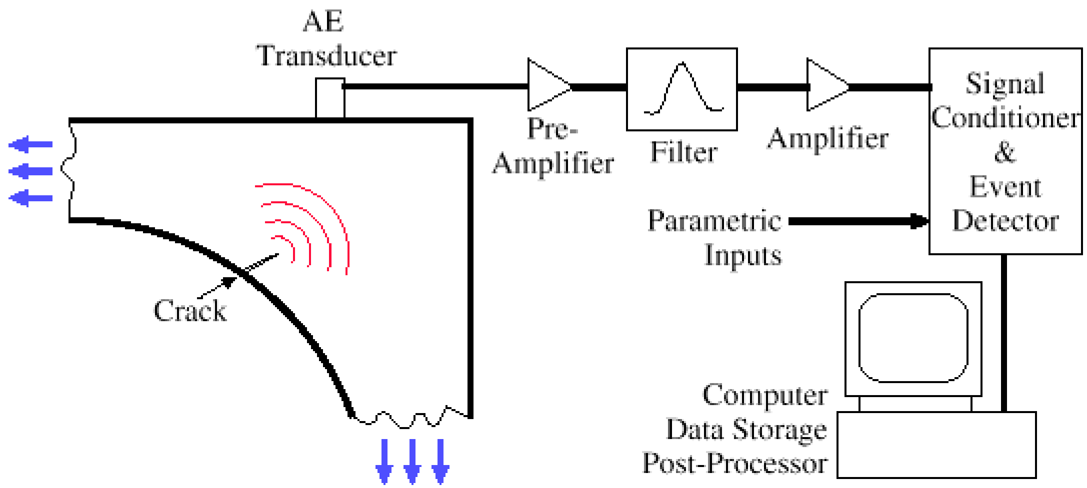
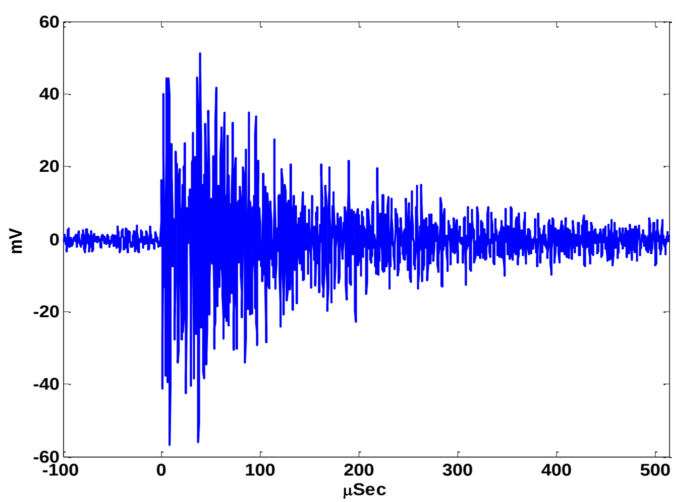


Fig. from [2]

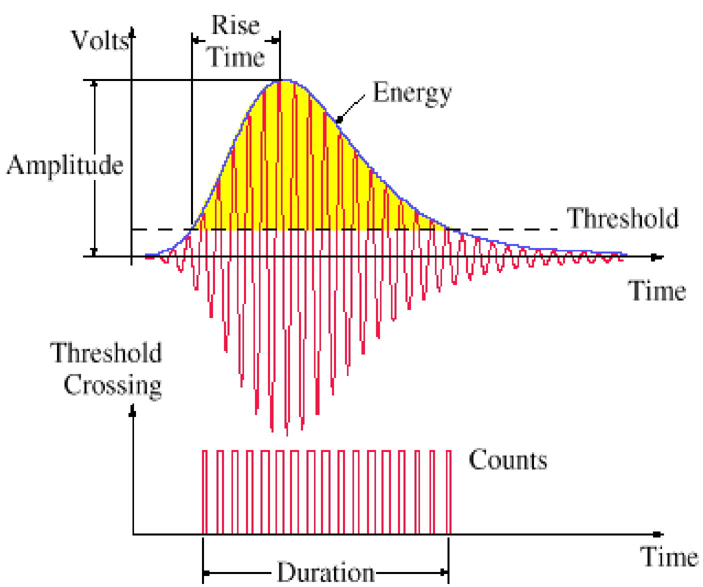


AE waveform from crack growth

Acoustic emissions are elastic stress waves generated by a rapid release of energy from localized sources within a material under stress [3].

- Passive technique (good for detecting damage as it accumulates)
- Global monitoring and localization capability
- Only good for detecting active defects
- Highly susceptible to noise

[2] Huang, M. et al., 1998. Using acoustic emission in fatigue and fracture materials research. *JOM*, 50(11), 1-14
[3] Mix, P.E., 2005. Introduction to nondestructive testing: a training guide, Wiley-Interscience



AE Features

- Amplitude
- Energy
- Rise time
- Counts (Threshold crossing)
- Frequency content
- Waveform shape

- Objective: To correlate AE parameters with fatigue crack growth parameters

Acoustic Emission

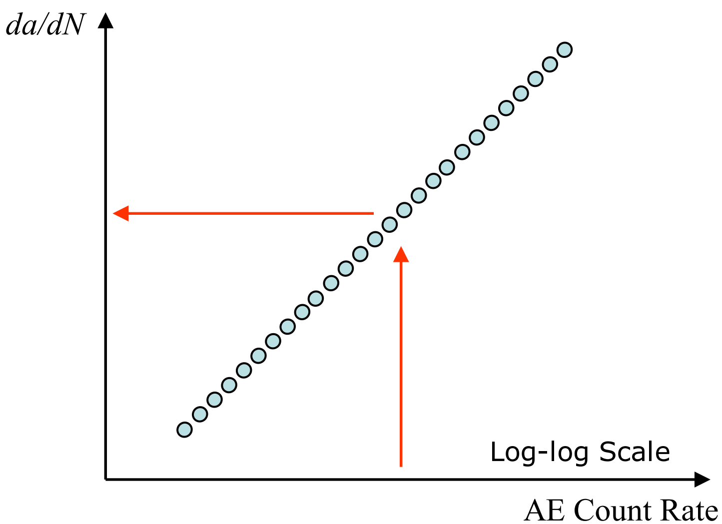
Amplitude
Energy
Rise time
Counts
Frequency
Waveform

correlation

Fatigue

Crack Growth Rate

ΔK=f(stress,crack size)

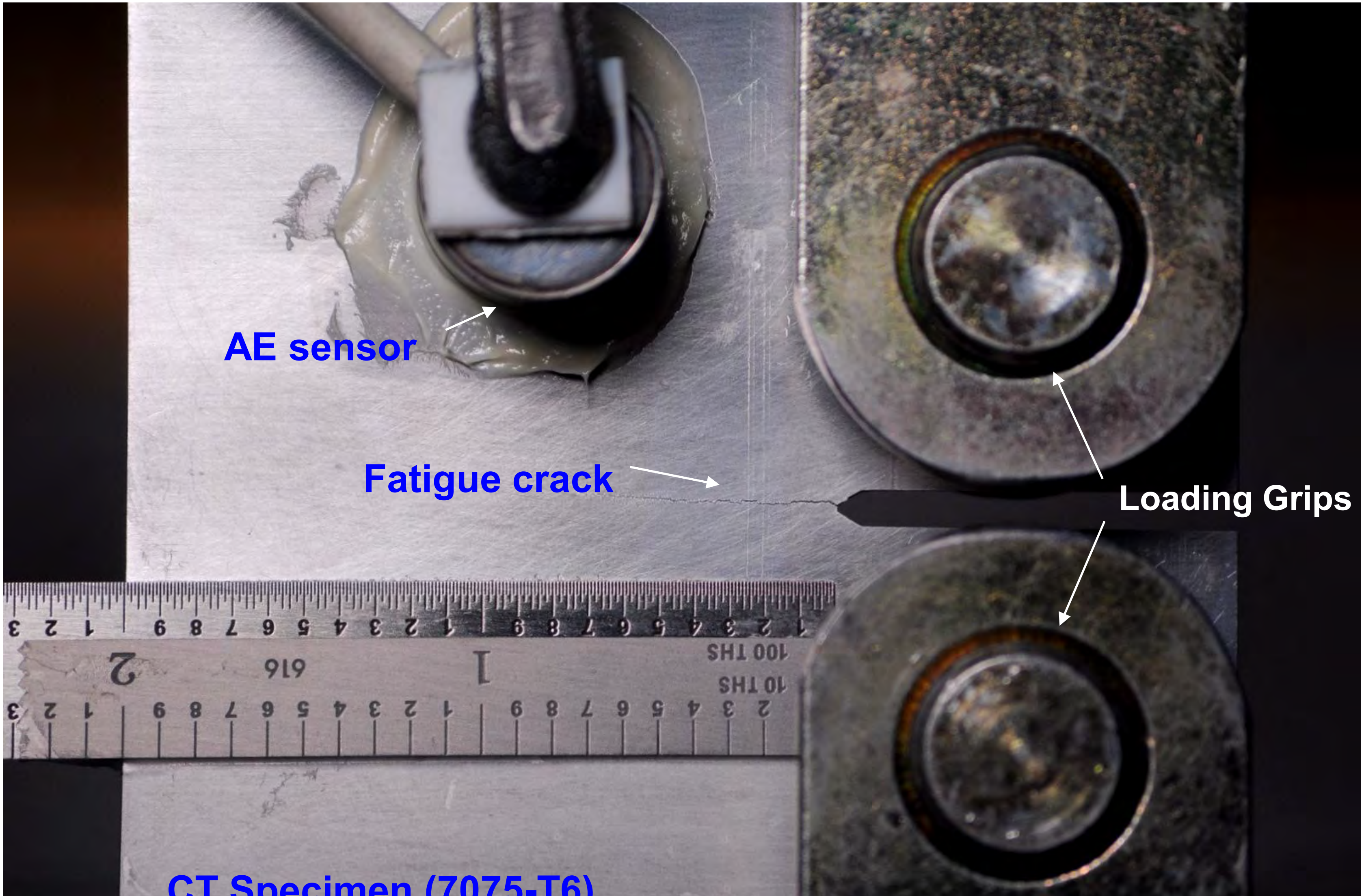


$$\log\left(\frac{da}{dN}\right) = \beta_1 \log\left(\frac{dc}{dN}\right) + \beta_2$$

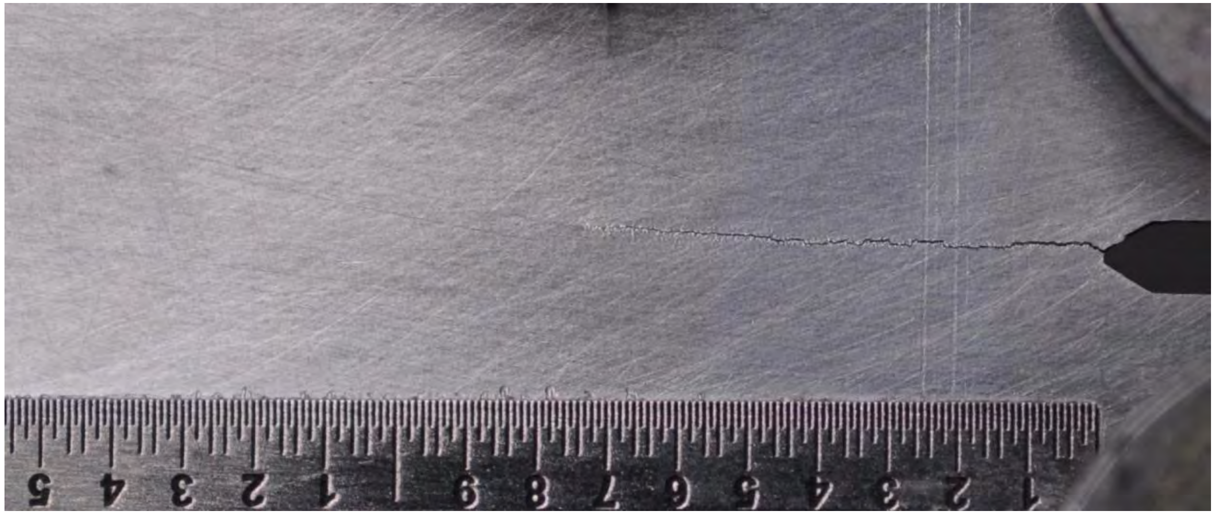
One can estimate *da/dN*, given β_1, β_2 and AE count rate

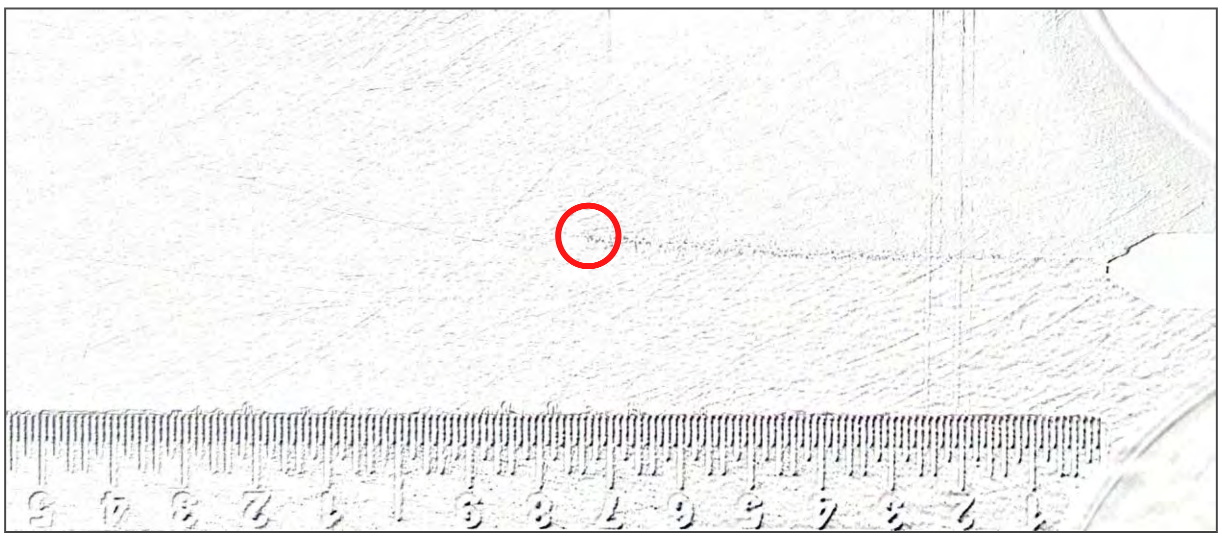
[4] Bassim, M.N., St Lawrence, S. & Liu, C.D., 1994. Detection of the onset of fatigue crack growth in rail steels using acoustic emission. *ENG FRACT MECH*, 47(2), 207-214.

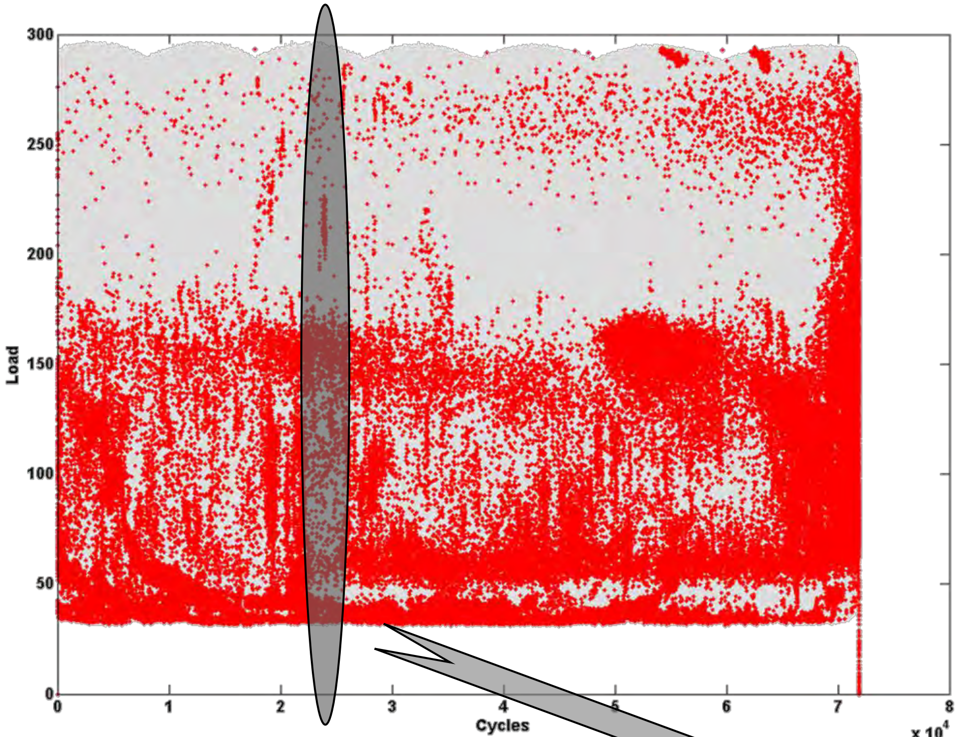
Experimental Procedure

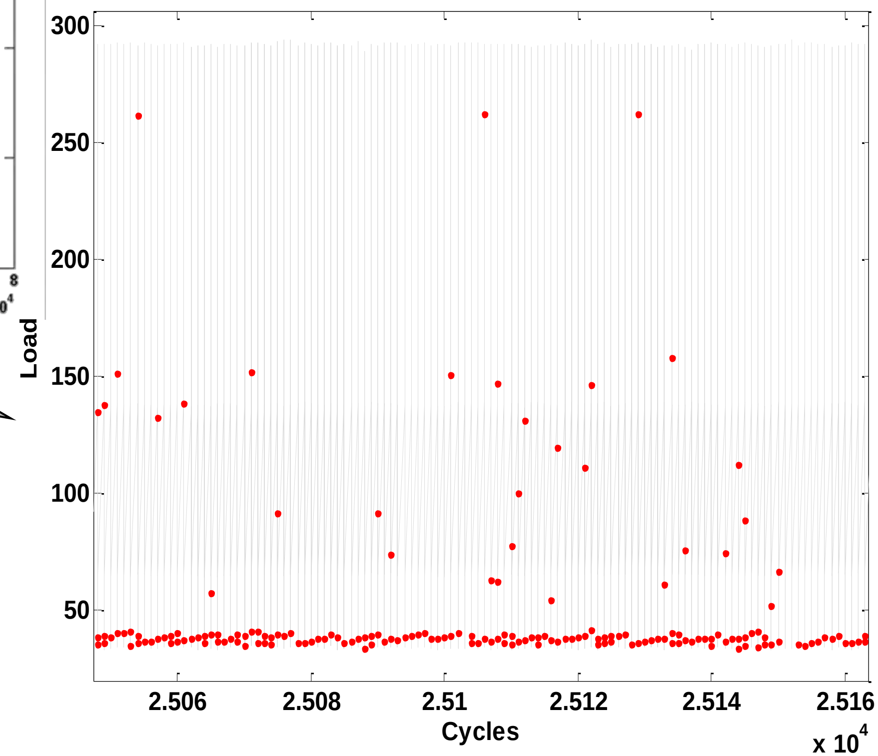


- [Crack Growth Clip]



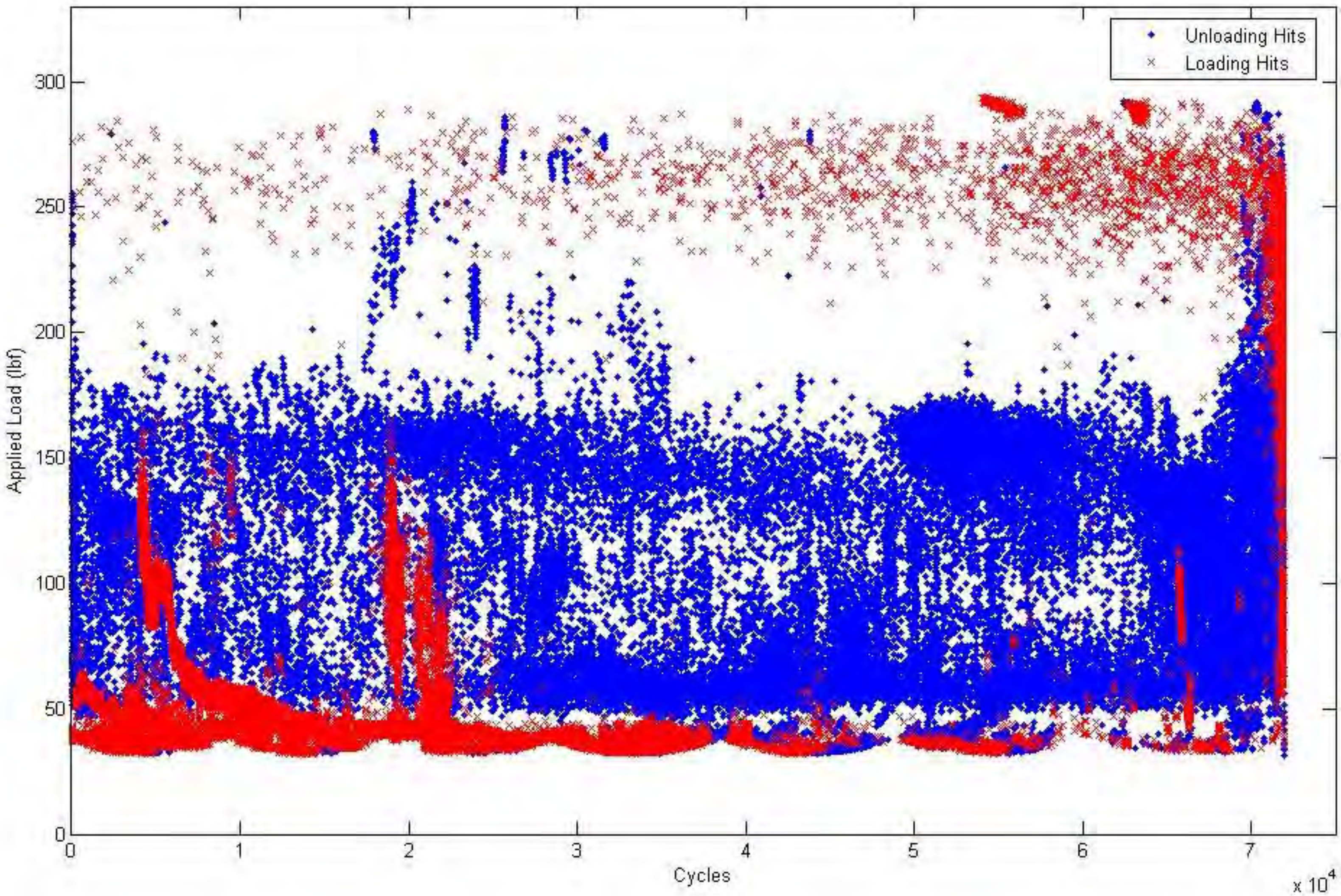




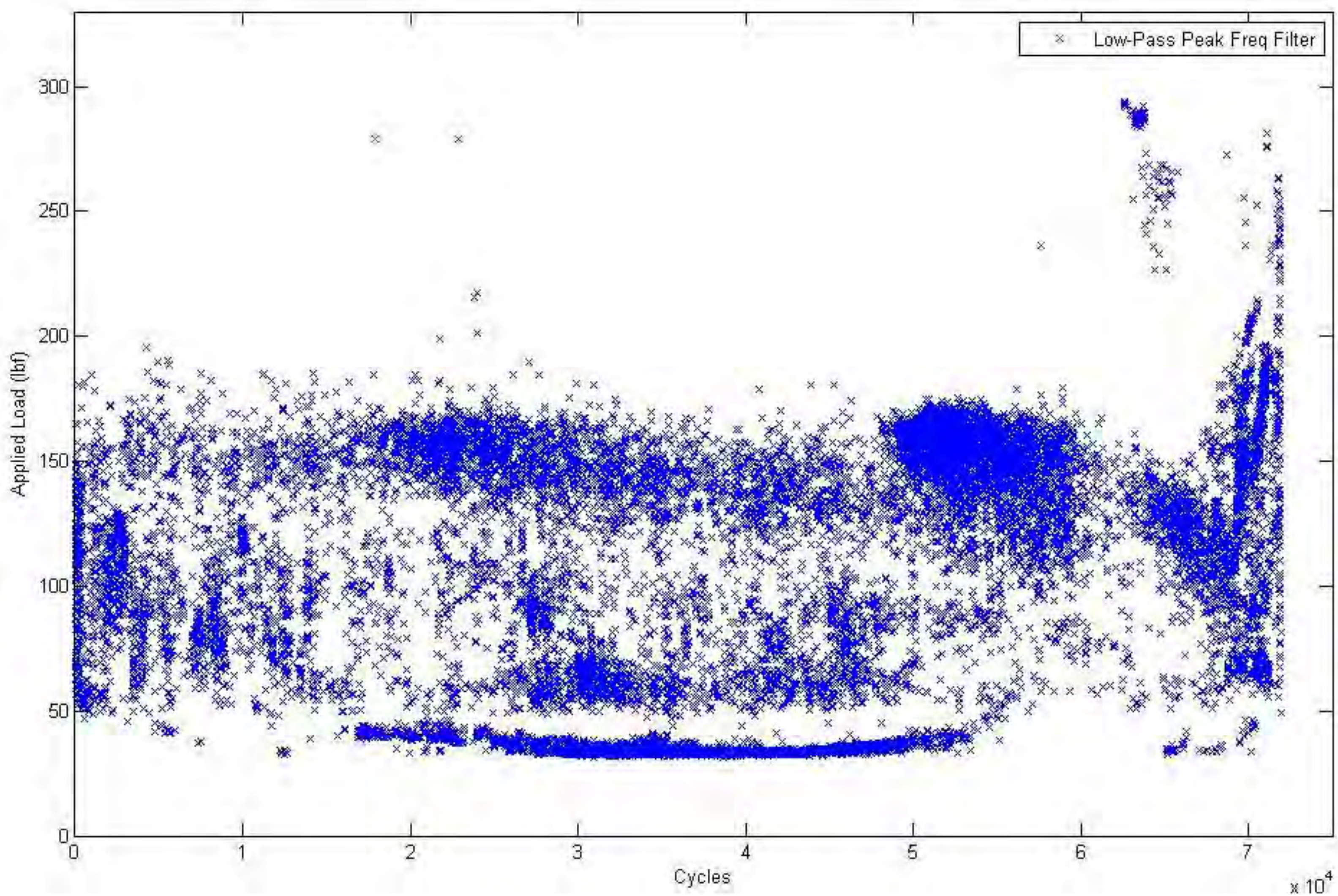


Noise (AE from any source other that crack growth)

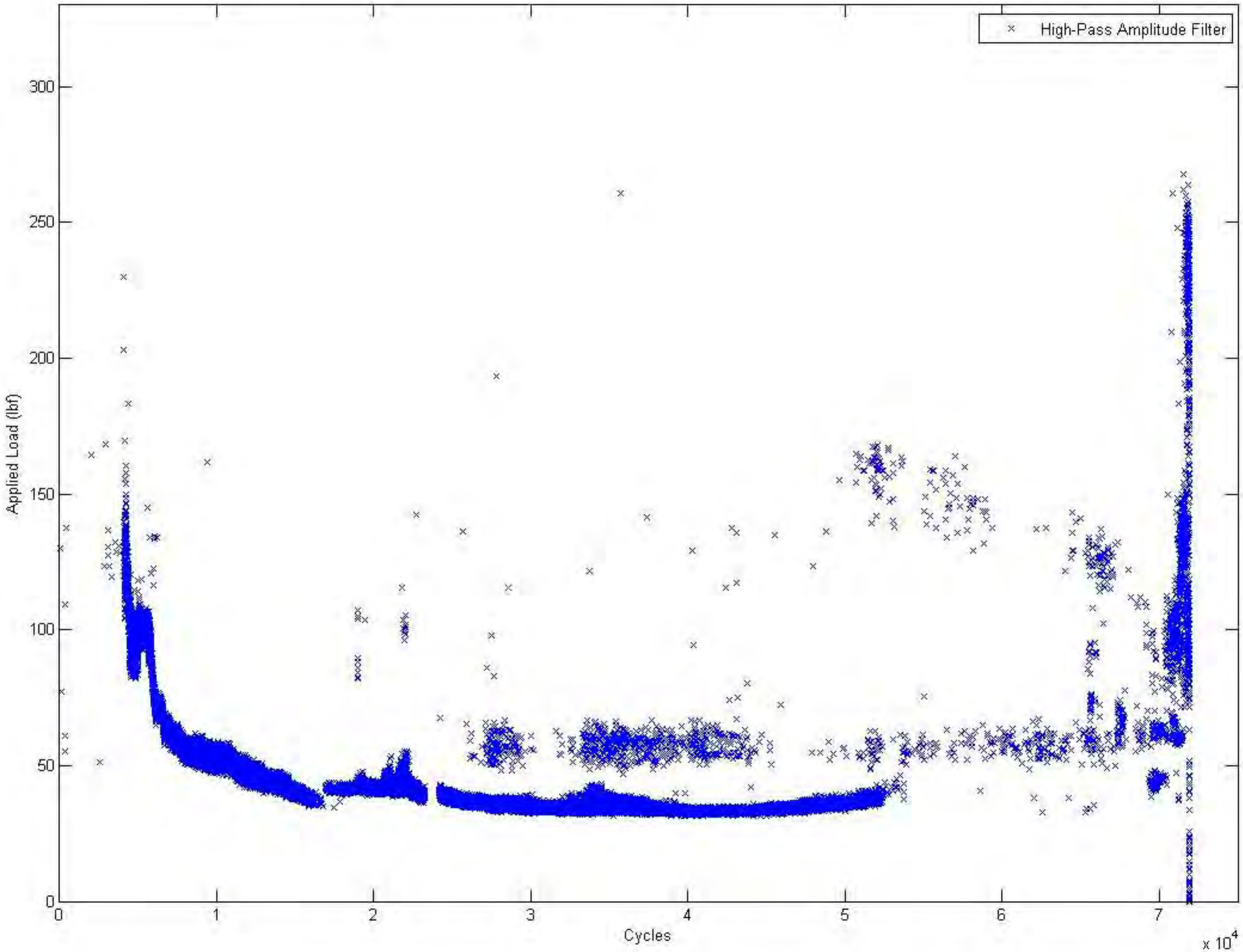
- Rubbing of crack surfaces
- Crack closure
- Grip noise
- Other active defects


Unloading Hits
Loading Hits
Applied Load (lbf)
Cycles
x 10^4
0
50
100
150
200
250
300
0
1
2
3
4
5
6
7

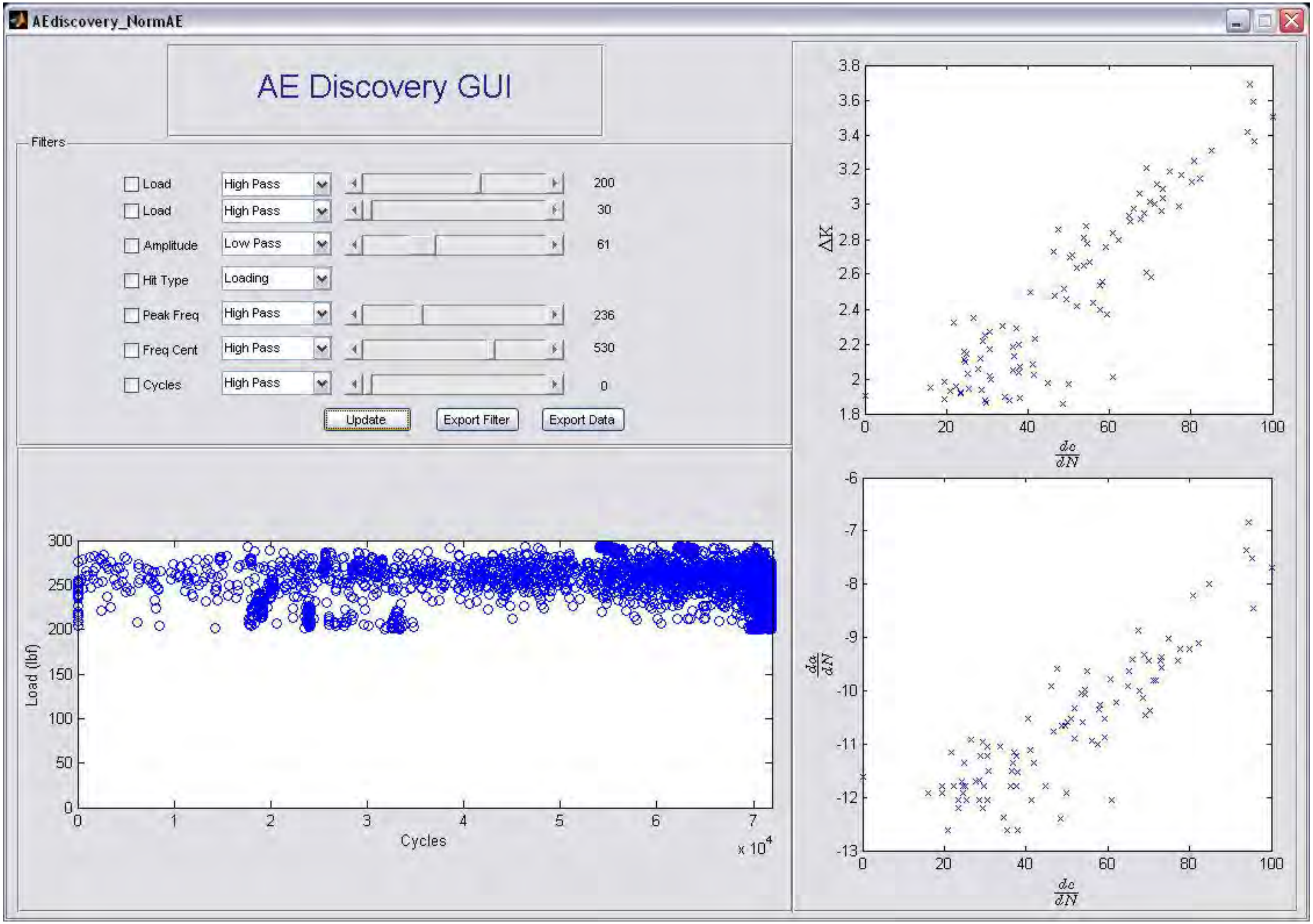
Experimental Procedure > Noise Filtration (Peak Freq.)



Experimental Procedure > Noise Filtration (Amplitude)



- AE Discovery Tool developed in MATLAB



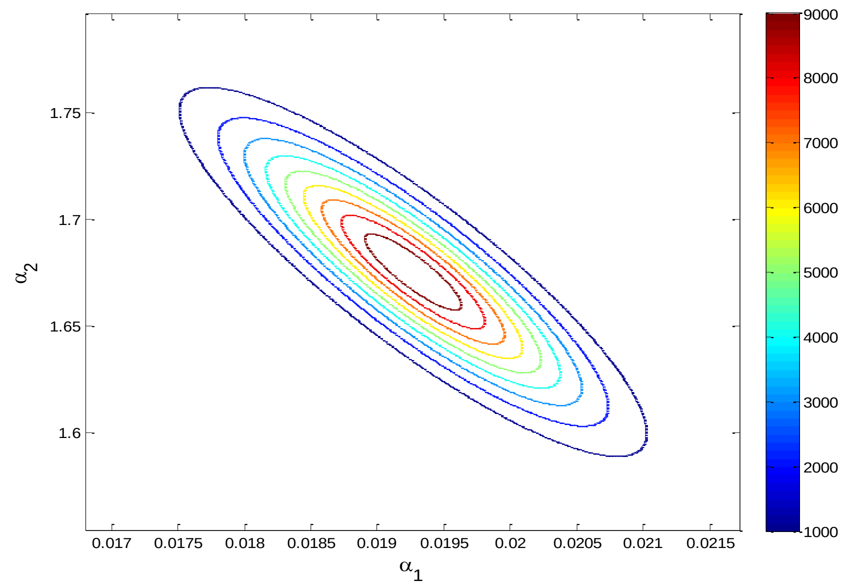
Model

$$Y = \beta_1 X + \beta_2 + \varepsilon$$
$$\varepsilon \sim \mathrm{Normal}(0,\sigma)$$

Likelihood

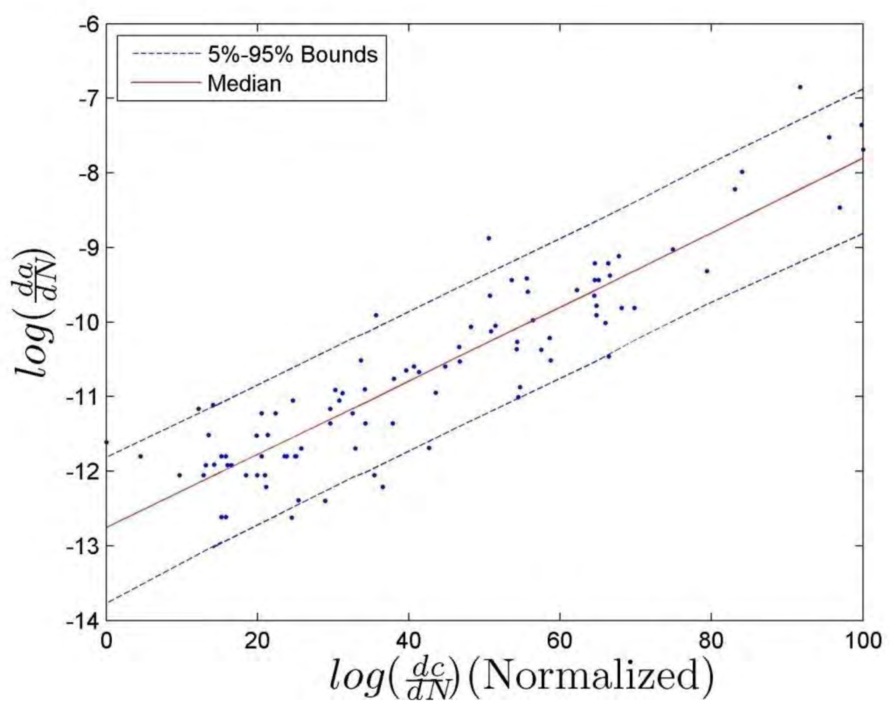
$$L(D \mid \underline{\theta}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{y_i - (\beta_1 x_i + \beta_2)}{\sigma}\right)^2}$$

Updating parameters via MCMC



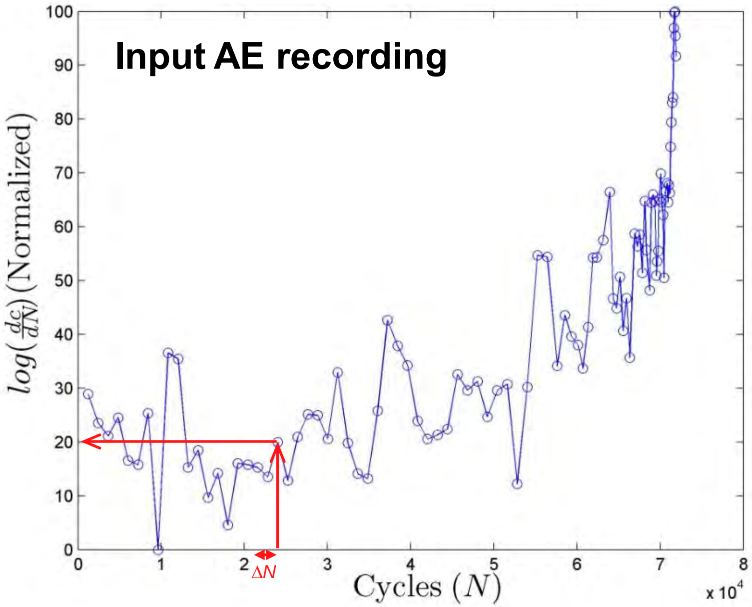
Integrating over parameters

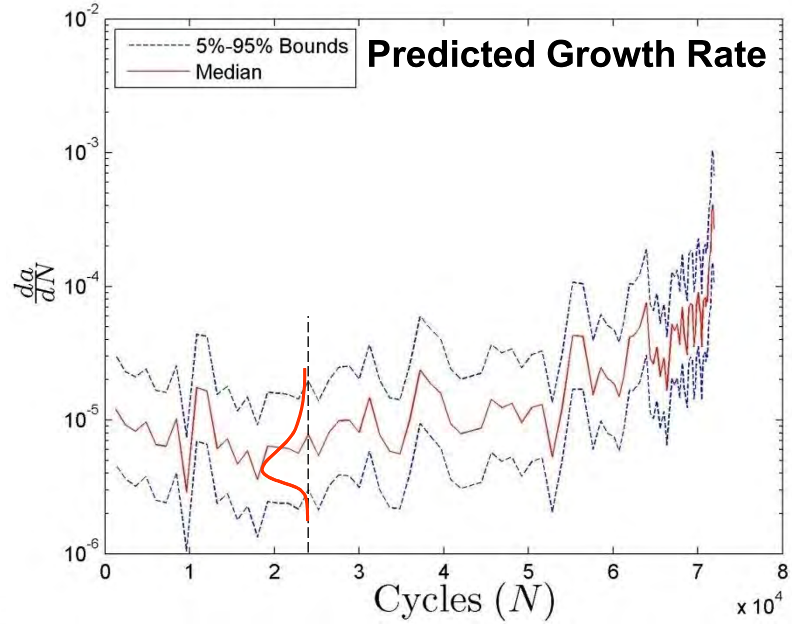
$$f(Y \mid X, D) = \iiint_{\underline{\theta}} f(Y \mid X, \underline{\theta})\,\pi(\underline{\theta} \mid D)\,d\underline{\theta}$$
$$\underline{\theta} = \{\beta_1, \beta_2, \sigma\}$$

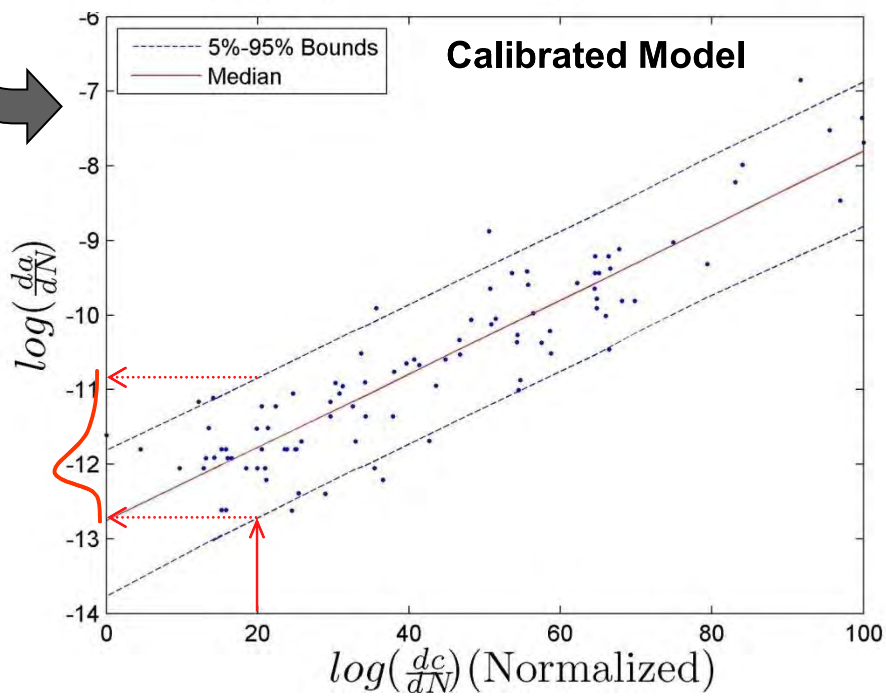


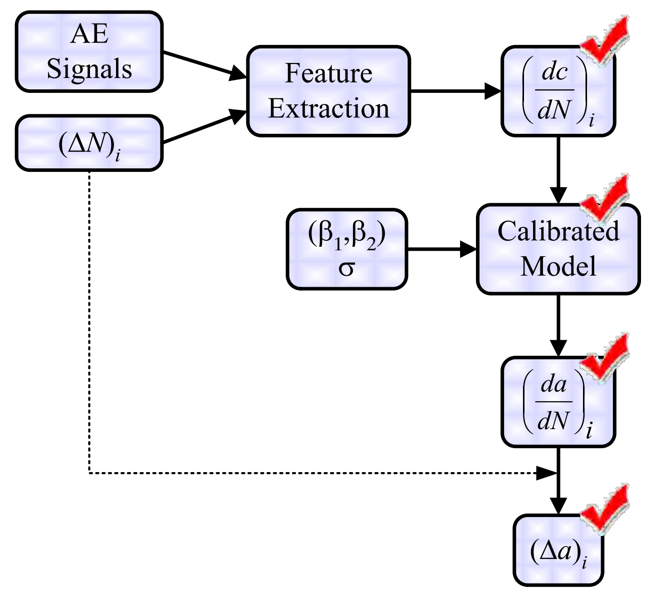
Probability density of *da/dN* For a given value of measured AE count rate and calibrated model parameters.

AE-based Crack Size Prediction

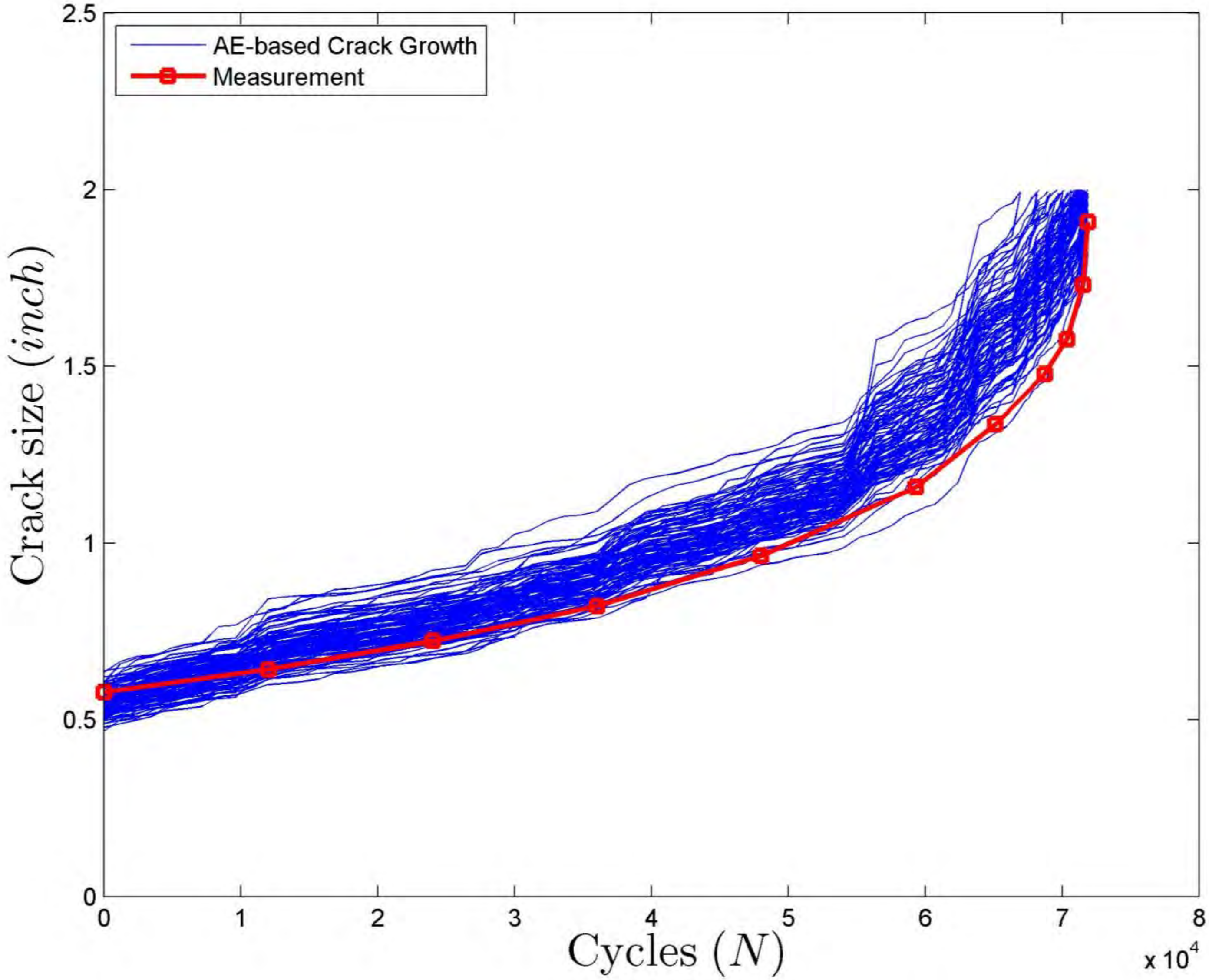




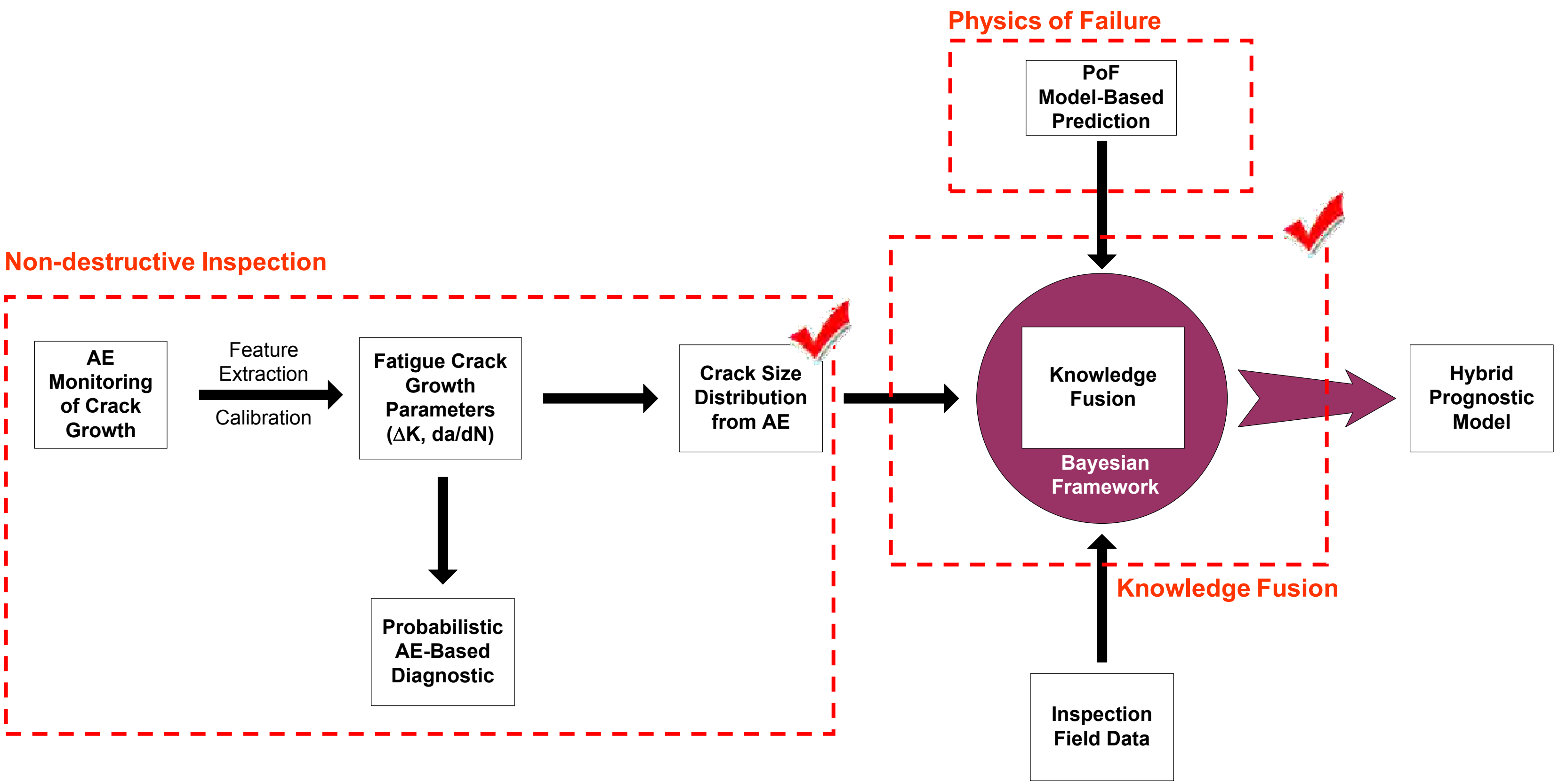


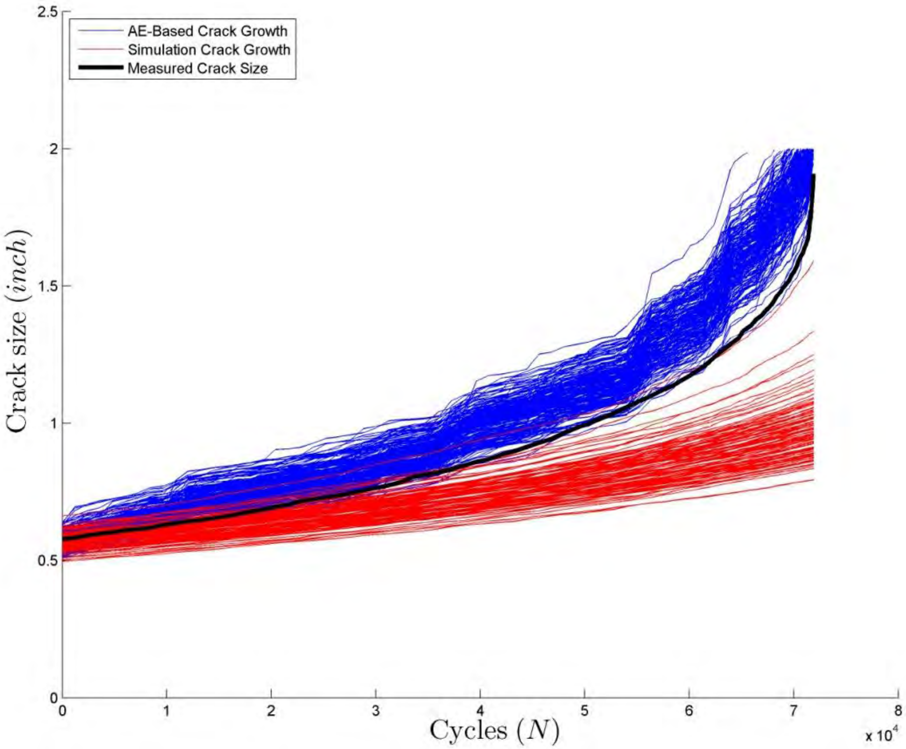


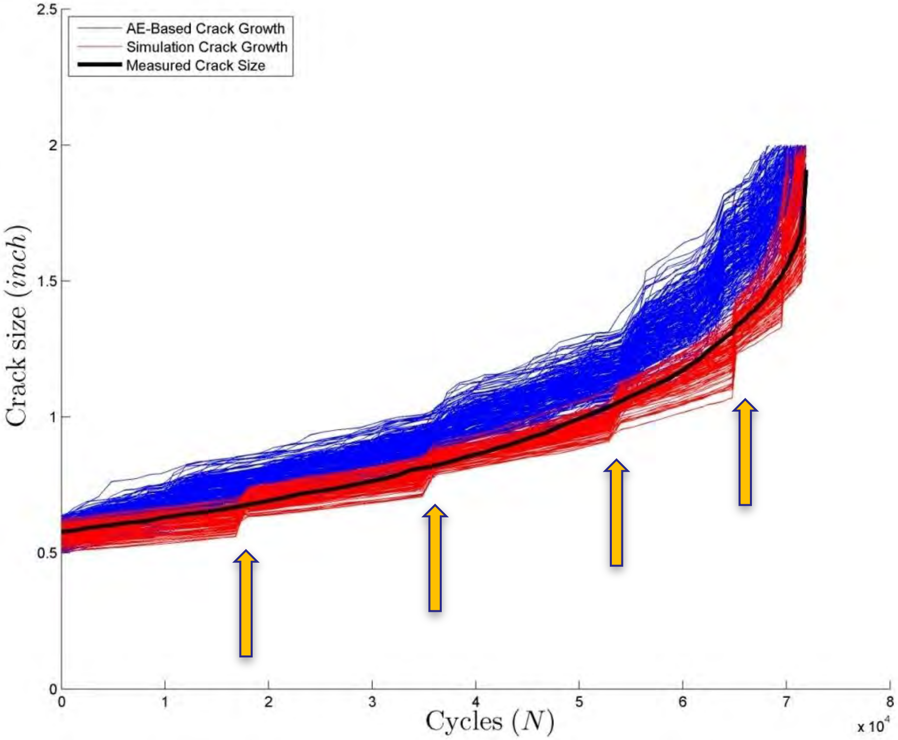
AE-based Crack Size Prediction

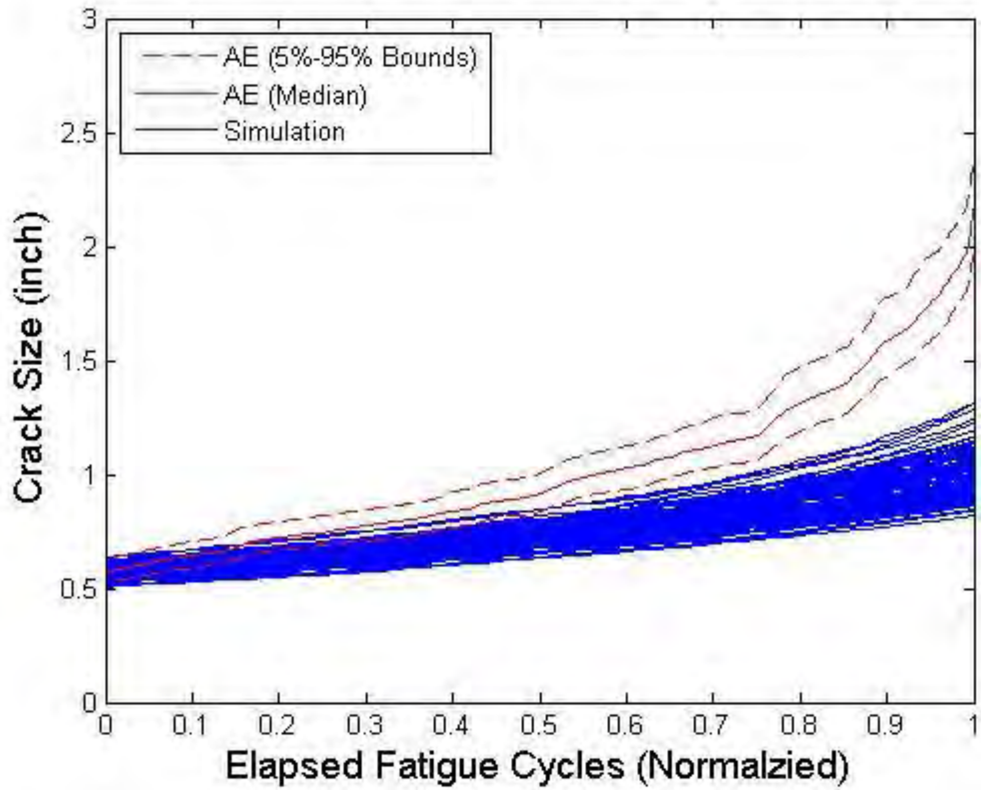


- Sources of Uncertainty:
  - Stochastic nature of the model
  - Initial crack distribution


Physics of Failure
PoF Model-Based Prediction
Non-destructive Inspection
AE Monitoring of Crack Growth
Feature Extraction
Calibration
Fatigue Crack Growth Parameters (ΔK, da/dN)
Crack Size Distribution from AE
Probabilistic AE-Based Diagnostic
Knowledge Fusion
Bayesian Framework
Knowledge Fusion
Hybrid Prognostic Model
Inspection Field Data


AE-Based Crack Growth
Simulation Crack Growth
Measured Crack Size
Crack size (inch)
Cycles (N)
x 10^4
0
0.5
1
1.5
2
2.5
0
1
2
3
4
5
6
7
8


AE-Based Crack Growth
Simulation Crack Growth
Measured Crack Size
Crack size (inch)
Cycles (N)
x 10^4
0
0.5
1
1.5
2
2.5
0
1
2
3
4
5
6
7
8



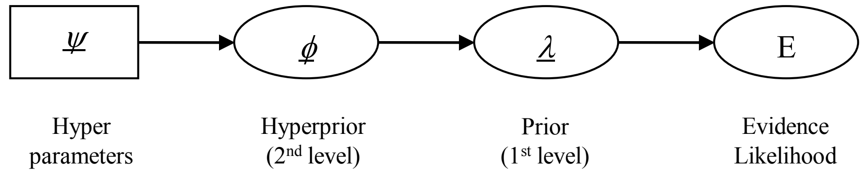
Evidence

Acoustic Emission: $D_{AE} = \left\{ D_{AE}^{(k)} \mid k = 1 \ldots N_e \right\}$

$$D_{AE}^{(k)} = \left\{ \left(x_k, E^{(k)} \right) \mid E^{(k)} \sim f_{E^{(k)}} \left(e^{(k)} \mid \underline{\delta}; x_k \right) \right\}$$

Simulation: $D_{SM} = \left\{ \left(x_j, y_{ij} \right) \mid i = 1 \ldots N_s, j = 1 \ldots N_x \right\}$

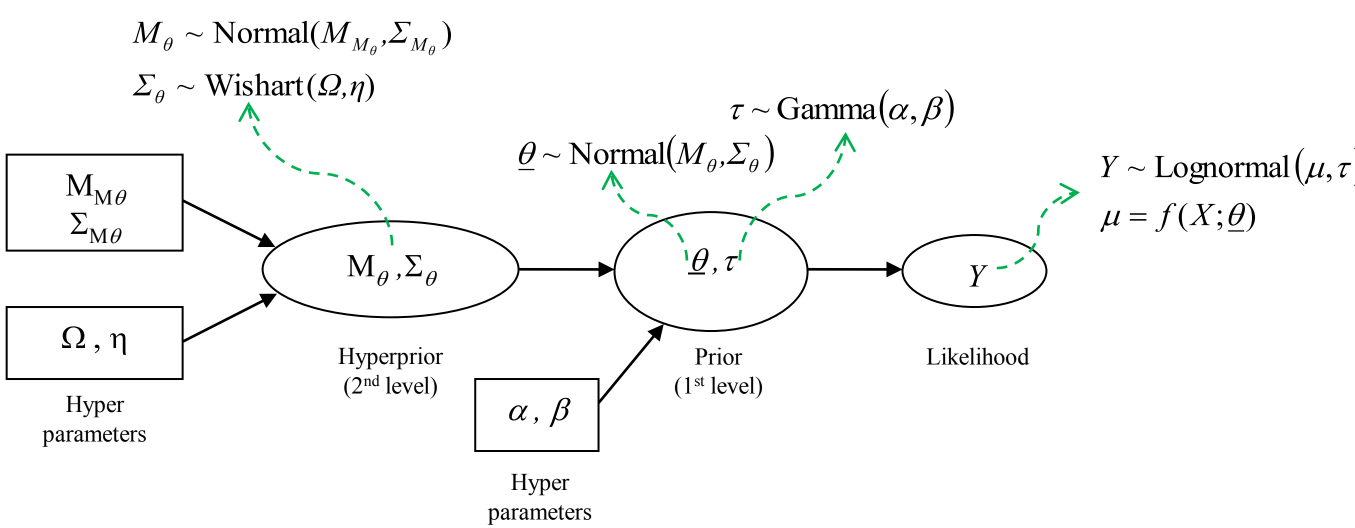
Hierarchical Bayesian Model



$\underline{\lambda} = \{\underline{\theta}, \sigma\}$ (First level prior)

$\underline{\phi} = \{ M_{\theta}, \Sigma_{\theta} \}$ (Second level prior)

$\underline{\Psi} = \{M_{M\theta}, \Sigma_{M\theta}, \Omega, \eta, \alpha, \beta\}$ (Vector of hyperparameters)



- The objective is to infer the model parameters from the simulation and the AE data.

$$p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, D_{AE}\right) = ?$$

$$p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, D_{AE}\right) = p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, D_{AE}^{(1)},\ldots,D_{AE}^{(N_e)}\right) = p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, E^{(1)},\ldots,E^{(N_e)}\right)$$

$$= \int_{e^{(1)}} f_{E^{(1)}}\left(e^{(1)}\right) p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, e^{(1)}, E^{(2)},\ldots,E^{(N_e)}\right) de^{(1)}$$

$$= \int_{e^{(1)}} \int_{e^{(2)}} f_{E^{(1)}}\left(e^{(1)}\right) . f_{E^{(2)}|E^{(1)}}\left(e^{(2)} \mid e^{(1)}\right) . p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, e^{(1)}, e^{(2)}, E^{(3)},\ldots,E^{(N_e)}\right) . de^{(2)} . de^{(1)}$$

$$\vdots$$

$$= \int_{e^{(1)},\ldots,e^{(N_e)}} f_{E^{(1)}}\left(e^{(1)}\right) . \prod_{k=2}^{N_e}\left(f_{E^{(k)}|E^{(k-1)}}\left(e^{(k)} \mid e^{(k-1)}\right) . \underline{p\left(\underline{\lambda},\underline{\phi} \mid D_{SM}, e^{(1)}, e^{(2)},\ldots,e^{(N_e)}\right)} . de^{(k)}\right)$$

The correlation between AE data at subsequent time instances is captured in the conditional PDF terms that appear in the above equations.

- Now using Bayes' rule:

$$p\left(\underline{\lambda},\underline{\phi}\mid D_{SM},e^{(1)},e^{(2)},\ldots,e^{(N_e)}\right)\propto p\left(D_{SM},e^{(1)},e^{(2)},\ldots,e^{(N_e)}\mid\underline{\lambda},\underline{\phi}\right)p\left(\underline{\lambda},\underline{\phi}\right)$$

$$=p\left(D_{SM}\mid\underline{\lambda},\underline{\phi}\right).p\left(e^{(1)}\mid\underline{\lambda},\underline{\phi}\right)\ldots.p\left(e^{(N_e)}\mid\underline{\lambda},\underline{\phi}\right).p\left(\underline{\lambda},\underline{\phi}\right)$$

Simplification based on the independence of D_{SM} and D_{AE} and conditional independence of $E^{(k)}$' s given the model parameters, i.e. $p\left(E^{(k_1)}|E^{(k_2)},\underline{\lambda},\underline{\varphi}\right)=p\left(E^{(k_1)}|\underline{\lambda},\underline{\varphi}\right),\forall k_1,k_2=1,\ldots,N_e$

$$=p\left(D_{SM}\mid\underline{\lambda}\right).p\left(e^{(1)}\mid\underline{\lambda}\right)\ldots.p\left(e^{(N_e)}\mid\underline{\lambda}\right).p\left(\underline{\lambda}\mid\underline{\phi}\right).p\left(\underline{\phi};\underline{\psi}\right)$$

$\underline{\phi}$ does not appear in the likelihood functions and by using the rules of conditional probability

At this level, each of the likelihood terms, $p(\,.|\,\underline{\lambda})$, can be easily calculated as follows:

$$p\left(D_{SM}\mid\underline{\lambda}\right)=\prod_{j=1}^{Nx}\prod_{i=1}^{Ns}f_Y\left(y_{ij}\mid\underline{\lambda};x_j\right)$$

$$p\left(e^{(k)}\mid\underline{\lambda}\right)=f_{Y_k}\left(e^{(k)}\mid\underline{\lambda};x_k\right)$$

- D_{SM} and D_{AE} are independent so we can do sequential updating: First update using D_{SM} , use the resulting posterior as the prior for updating with D_{AE}

- For D_{AE} , discretize the distribution of $E^{(k)}$'s and treat each resulting point as regular evidence. Perform the updating but weigh the resulting posterior using an appropriate weight calculated from the conditional distribution $f_{E^{(k)}|E^{(k-1)}}\left(e^{(k)} \mid e^{(k-1)}\right)$
- Bayesian updating at each step is performed via MCMC simulation
  - We use WinBUGS software to find the posterior
  - Weight calculation is performed in MATLAB
- Large computation time for large data sets and lower discretization error.

- A case study of using Bayesian fusion technique for integrating information from multiple sources in a structural health management problem was presented
- The simulation data was first used to find the model parameters and then, as crack size estimates from AE became available, the model parameters were updated in light of the new evidence.
- The mathematical formulation of the problem as well as the setup of the Bayesian inference solution was given.
- The solution includes treatment of 'uncertain' evidence and also takes into account the correlation between AE observations.
- The resulting equations should be solved numerically. Efforts are still under way to provide an efficient computational solution to this problem

Thanks you!

Questions? Comments?