Assessing Reliability Using Developmental and Operational Test Data

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Agenda

- Background
- Overarching Model Framework
- Reliability growth in Developmental Testing (DT)
- Combing DT and Operational Testing (OT) Results
- Performance Comparisons
- Conclusions and Future Work

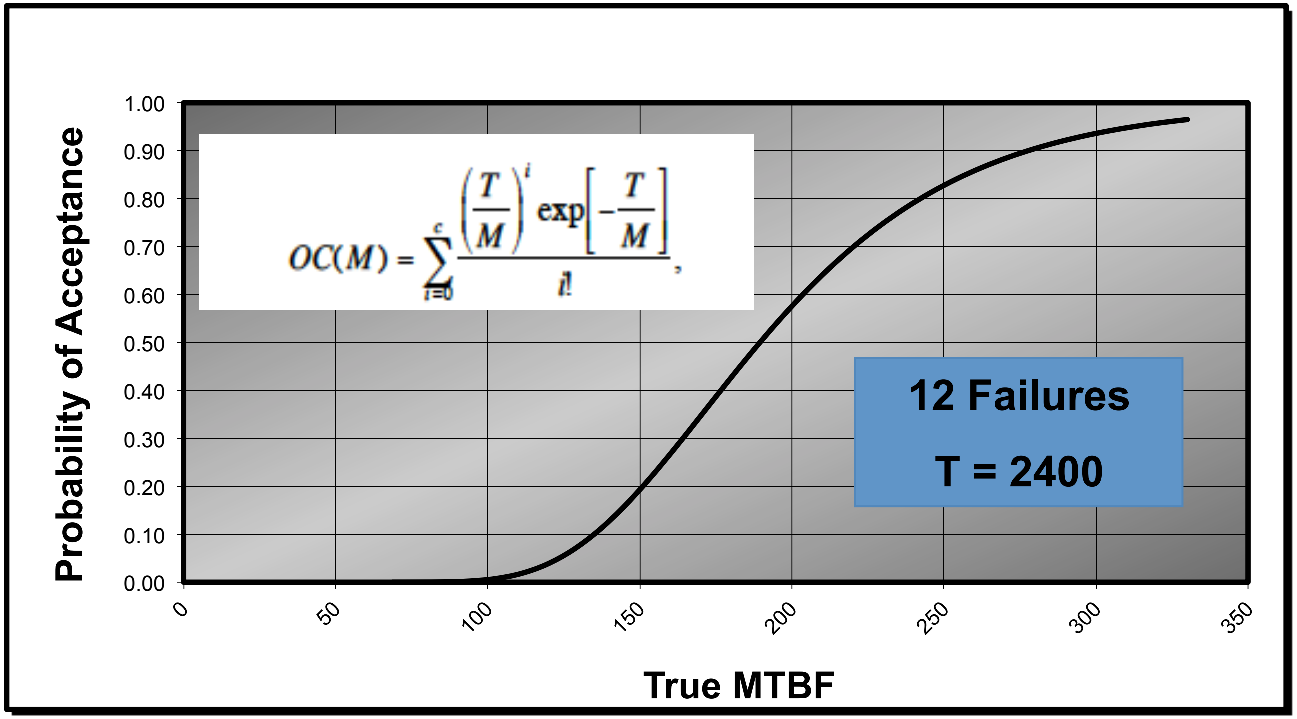
Background

- New military product development generally contains developmental reliability growth testing as design matures to final state

- Developmental testing environment may not completely represent operational usage environment
  - Operators are different
  - Loads and stresses may be different

- Reliability currently assessed in single operational test
  - Final mature configuration
  - Generally short and expensive tests

Problems with Current Operational Assessment



M = true but unknown reliability of the system and c = maximum number of allowable failures T = total demonstration test length

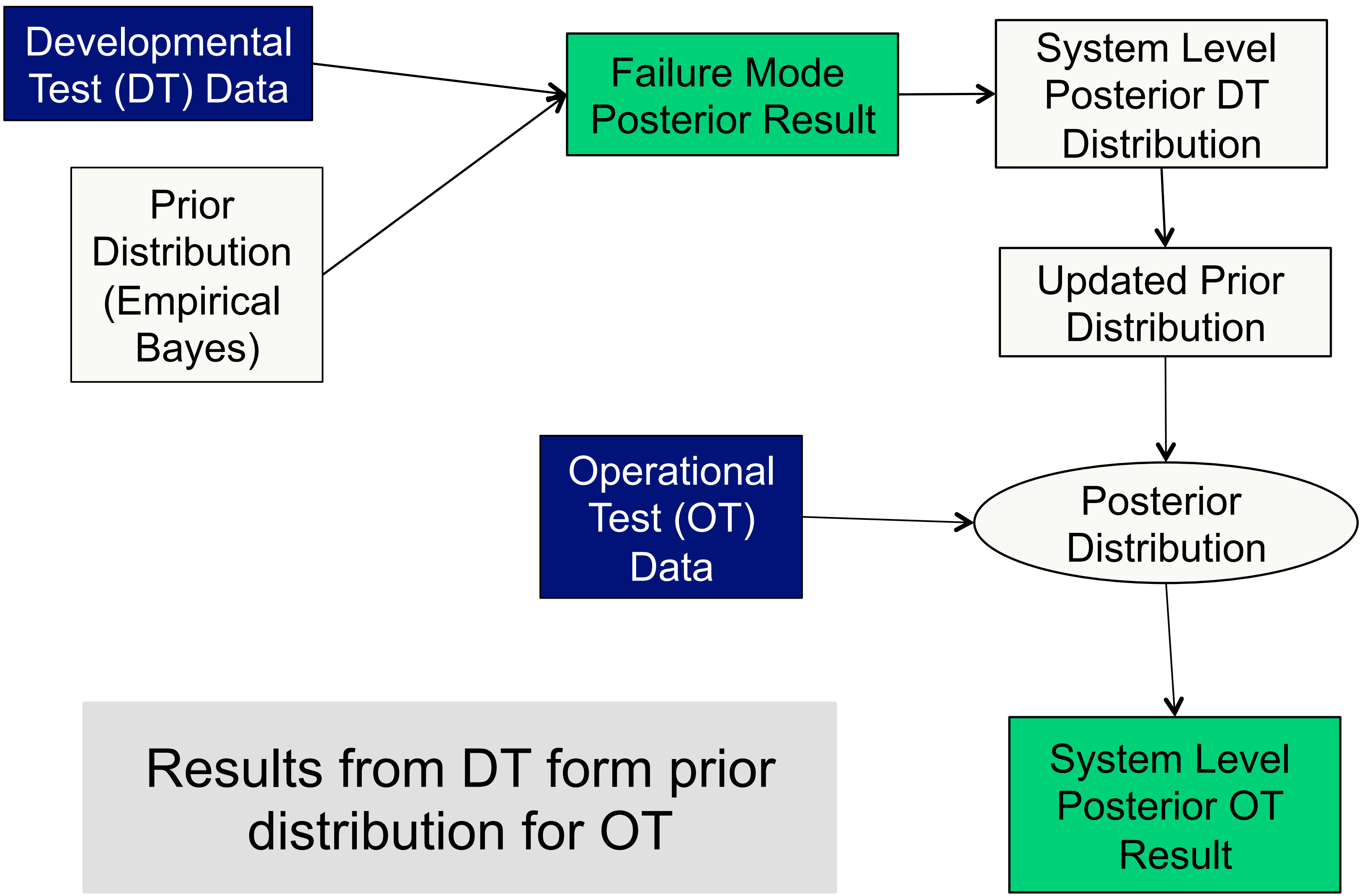
- Short test lengths can lead to “flat” Operating Characteristic (OC) curves. This often results in test plans in which no failures or a single failure are allowable.
- Resource constraints and technology maturity may make demonstration infeasible

Proposed Alternative Assessment

- Utilize data available from developmental testing within Bayesian framework
  - Account for reliability growth during development
  - Account for differences in test environment/conduct

- Benefits include:
  - Narrower probability intervals
  - Reduced testing requirements, lower costs, etc.
  - Can use additional data sources in reliability assessment

Overarching Framework



Results from DT form prior distribution for OT

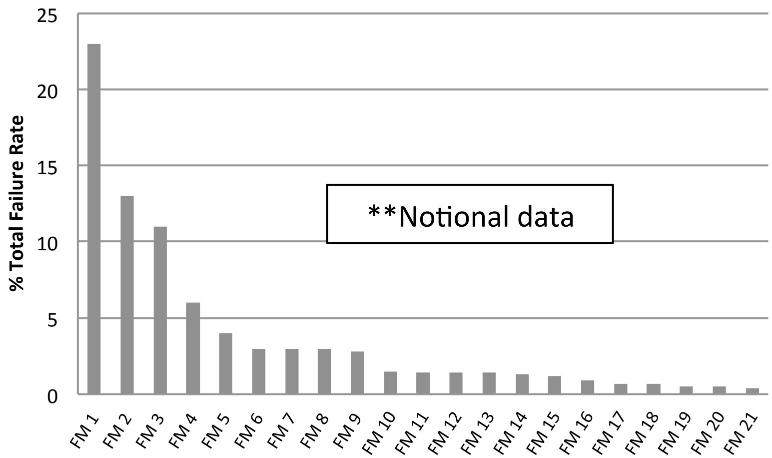
Likelihood for DT Reliability Growth

- Accounts for arbitrary corrective action strategy
- For each failure mode, i, in the system assume:
  - Failure intensity is constant before and after corrective action
  - *n* failures in demonstration test time T_{DT} with n_1 occurring before corrective action
  - Corrective action at time *v* with Fix Effectiveness Factor (FEF) *d*
- Likelihood given by

$$l\left(t_{i,1}, t_{i,2}, \ldots, t_{i,n_i}, n_i, n_{i,1} \mid \lambda_i\right) = \left(1-d_i\right)^{n_i - n_{i,1}} \lambda_i^{n_i} \exp\left(-\lambda_i\left[v_i + \left(1-d_i\right)\left(T_{DT} - v_i\right)\right]\right)$$

Likelihood allows for arbitrary reliability growth

Choice of Prior Distribution on Failure Intensity



$$p(\lambda) = \frac{\lambda^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} \exp\left(-\frac{1}{\beta}\lambda\right)$$

Provides inherent connection between failure modes

- Assumes failure mode failure intensities realized from common Gamma(α, β) distribution
- Gamma Follows “vital-few, trivial-many” construct
- Can use Empirical Bayes to estimate parameters

$$p(\lambda_i \mid n_i) = \frac{\lambda_i^{\alpha+n_i-1}}{\Gamma(\alpha+n_i)\left[\frac{1}{\beta}+v_i+(1-d_i)(T_{DT}-v_i)\right]^{-(\alpha+n_i)}} \exp\left[-\lambda_i\left(\frac{1}{\beta}+v_i+(1-d_i)(T_{DT}-v_i)\right)\right]$$

System Level Result

- System level estimate:
  - Summing over *m* total failure modes
  - Take limit as *m* becomes large
  - n number of failures for each of the m observed failure modes, i.
- For *m* observed modes, system level mean is

$$E[\lambda_s] = \sum_{i=1}^{m} \left(\frac{(1-d_i)n_i}{\frac{1}{\beta} + v_i + (1-d_i)(T - v_i)} \right) + \left(\frac{\lambda_B}{1+\beta T} \right)$$

Observed

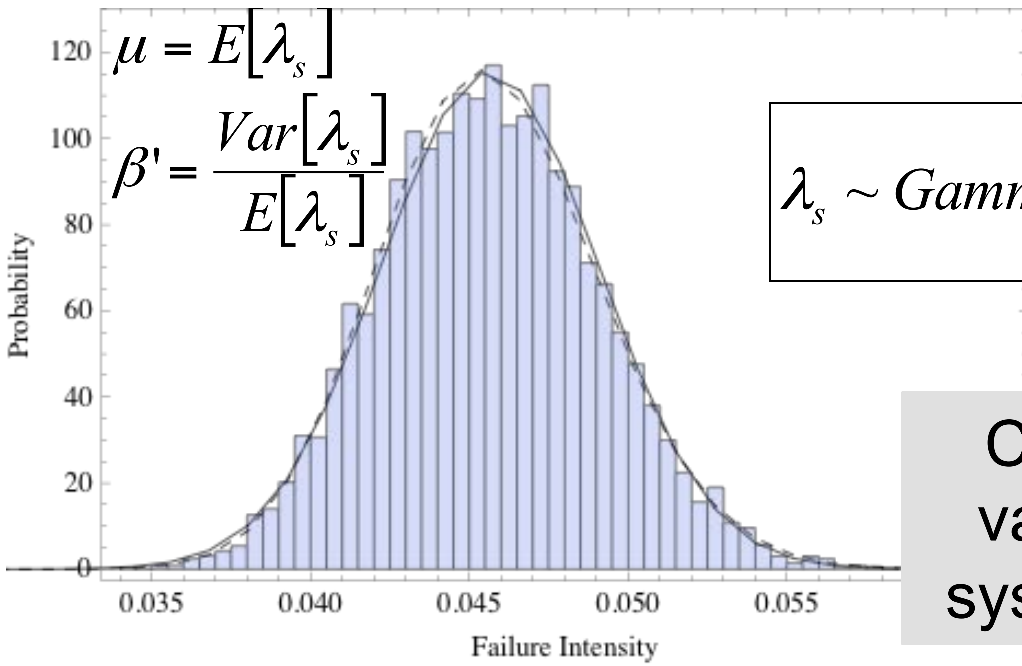
Unobserved

where $\lambda_{\mathrm{B}} = m\alpha\beta \equiv$ prior mean

Estimate includes contribution from unobserved modes

System Level Posterior Distribution

- Posterior can be simulated to determine approximate distribution
  - Exactly Gamma if corrective actions are delayed
  - Gamma approximation reasonable for arbitrary corrective action strategies



$$\mu = E[\lambda_s]$$

$$\beta' = \frac{Var[\lambda_s]}{E[\lambda_s]}$$

$$\boxed{\lambda_s \sim Gamma\left(\alpha' = \frac{\mu}{\beta'}, \beta'\right)}$$

Can use mean and variance to develop system level posterior

Incorporating Operational Data

- Generally have increased failure intensity in operational environment
  - DT conditions more benign, human factors, etc.
- Define MTBF as reciprocal of mode failure intensity λ
- Assume 100γ% decrease (degradation factor) in instantaneous MTBF (1/λ) such that

$$\lambda_{DT} = (1-\gamma)\lambda_{OT}$$

- Transformed prior found using properties of Gamma distribution

$$\lambda_{OT} \mid \gamma = \frac{\lambda_{DT}}{(1-\gamma)} \sim Gamma\left[\alpha', \frac{\beta'}{(1-\gamma)}\right]$$

Scaled prior accounts for reliability degradation

Marginal Posterior Distribution

- Assume n_2 failures in operational test length T_2
- Treats degradation factor γ as nuisance parameter
- Marginal posterior development
  - Compute joint posterior
  - Compute marginal distribution by integrating over nuisance parameter

$$l\left(t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} \mid \lambda_{OT}\right) = \lambda_{OT}^{n_{OT}} \exp\left(-\lambda_{OT} T_{OT}\right)$$

$$p\left(\lambda_{OT} \mid n_{OT}\right) = \int_{\Gamma} \frac{p(\gamma)\, p\left(\lambda_{OT} \mid \gamma\right) l\left(t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} \mid \lambda_{OT}\right)}{\iint_{\Lambda,\Gamma} p(\gamma)\, p\left(\lambda_{OT} \mid \gamma\right) l\left(t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} \mid \lambda_{OT}\right) \partial \lambda_{OT}\, \partial \gamma} \partial \gamma$$

- Use Beta prior distribution for γ

Marginal posterior probabilistically accounts for degradation

OT Posterior Assessment

- Posterior mean is scaled mean for Gamma distribution

$$E[\lambda_s \mid n] = \left(\frac{\frac{\mu}{\beta'} + n_2}{\frac{1}{\beta'} + T_2} \right) \frac{{}_2F_1\left[\frac{\mu}{\beta'} + n_2 + 1, a, a + b + \frac{\mu}{\beta'}, \frac{\frac{1}{\beta'}}{\frac{1}{\beta'} + T_2} \right]}{{}_2F_1\left[\frac{\mu}{\beta'} + n_2, a, a + b + \frac{\mu}{\beta'}, \frac{\frac{1}{\beta'_1}}{\frac{1}{\beta'} + T_2} \right]}$$

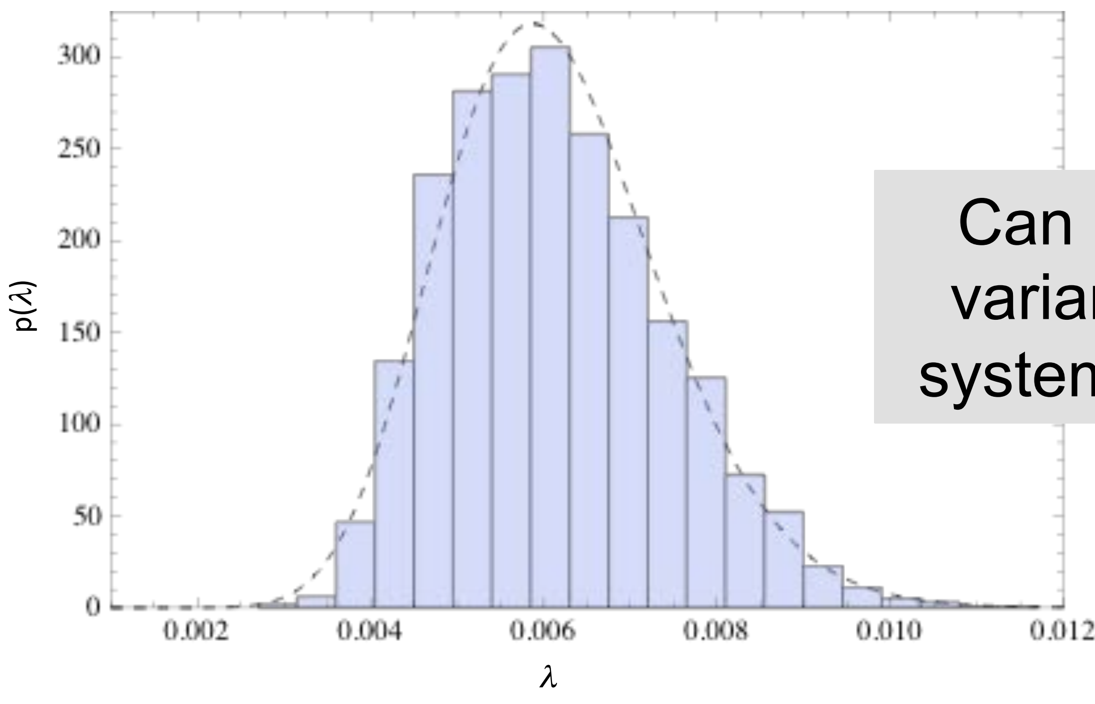
where μ, β' defined as on slide 9

Standard posterior mean for Gamma

Ratio of Hypergeometric functions accounts for DT/OT degradation

System Level OT Posterior Distribution

- Exact system-level posterior can be simulated using Markov Chain Monte Carlo methods
- Posterior well approximated with Gamma distribution
- Use approximate Gamma to develop probability intervals

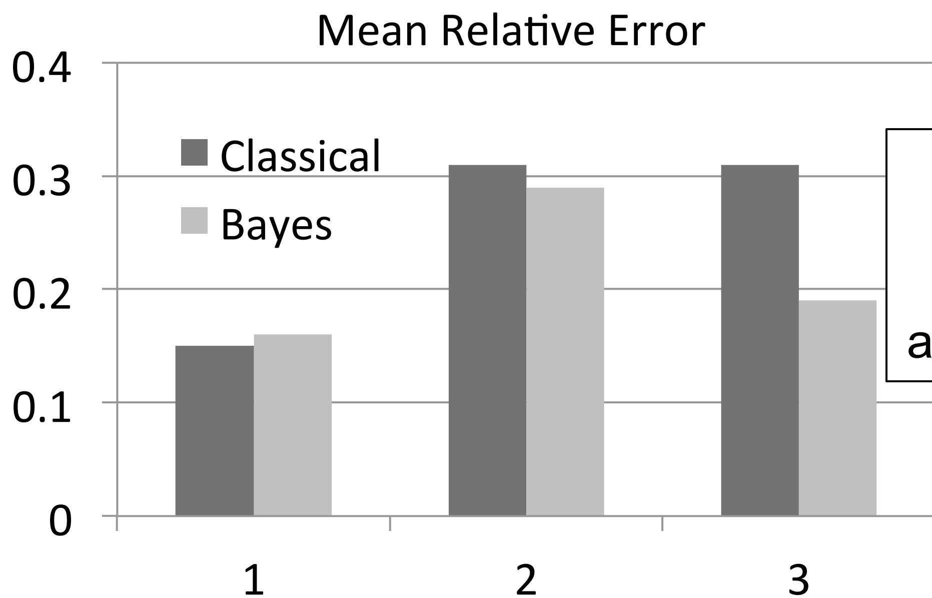


Can use mean and variance to develop system level posterior

Performance Comparisons

Case	Initial DT MTBF	DT Length	OT Length
1	100	1000	2000
2	400	2000	2000
3	100	2000	500

Simulation developed to examine relative error between model estimate and "true" value



Full error distribution comparisons available in paper

Bayesian approach performs better than current methods

Conclusions/Future Work

- Use of reliability data from developmental testing provides additional information that increases performance of overall reliability estimate

- Bayesian probabilistic approach provides flexibility
  - Can utilize multiple information sources
  - Can include additional sources of uncertainty

- Current/future efforts include:
  - Modeling uncertainty on FEF values
  - Developing prior information from additional data sources
  - Analogous results for discrete systems

