Advances in Science-Based Reliability Engineering

Presented to

FM Global

Norwood, MA

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TOPICS OF THIS SEMINAR

- Definitions
- Probabilistic Physics of Failure
- Accelerated Testing
- Science-Based Reliability Methods
- Conclusions

FORMAL PROBABILISTIC DEFINITION OF RELIABILITY

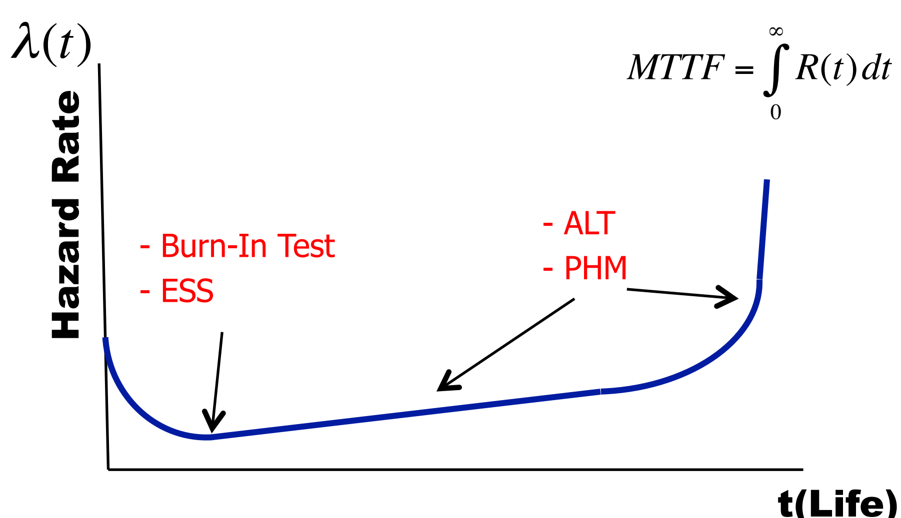
- Failure; In materials are caused by excessive degradation and damage due to underlying processes (failure mechanisms)
- Assumption: *Time* is an *aggregate* index representing causes of failure
- Reliability: The ability of an item (product, system, etc.) to operate under designated operating conditions for a designated period of time, number of cycles or stress

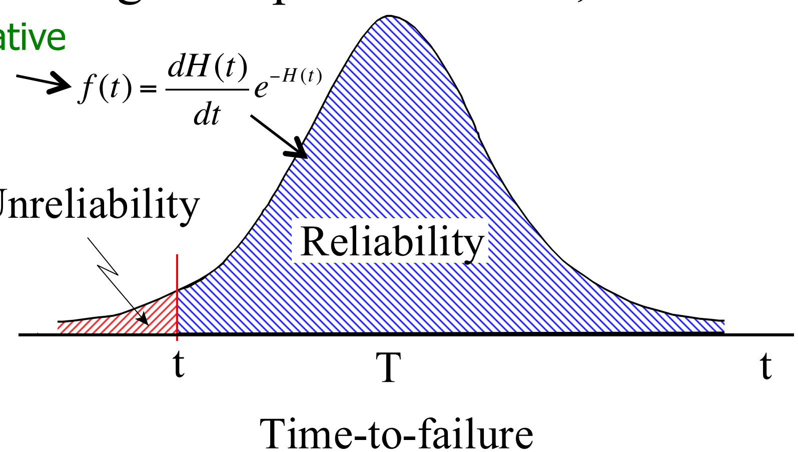
Cumulative Hazard

$$R(t) = e^{-\int_0^t \lambda(u)\,du} = e^{-H(t)}$$

$$f(t) = \frac{dH(t)}{dt} e^{-H(t)}$$

$$MTTF = \int_0^\infty R(t)\,dt$$





t = mission

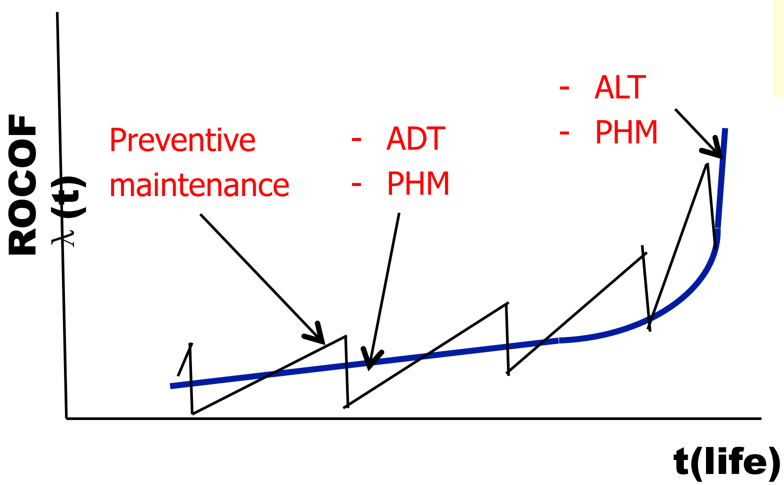
T = time-to-failure, cycle-to-failure

REPAIR, MAINTENANCE AND RENEWAL PROCESS

- Rate of Occurrence of Failure (ROCOF) instead of hazard rate is applied
- Reliability / availability is measured

by a non-homogeneous Poisson process:

$$\Pr\left[N(t_2) - N(t_1) = n\right] = \frac{\left[\int_{t_1}^{t_2} \lambda(t)\,dt\right]^n e^{-\left[\int_{t_1}^{t_2} \lambda(t)\,dt\right]}}{n!}$$



Cumulative Intensity Function

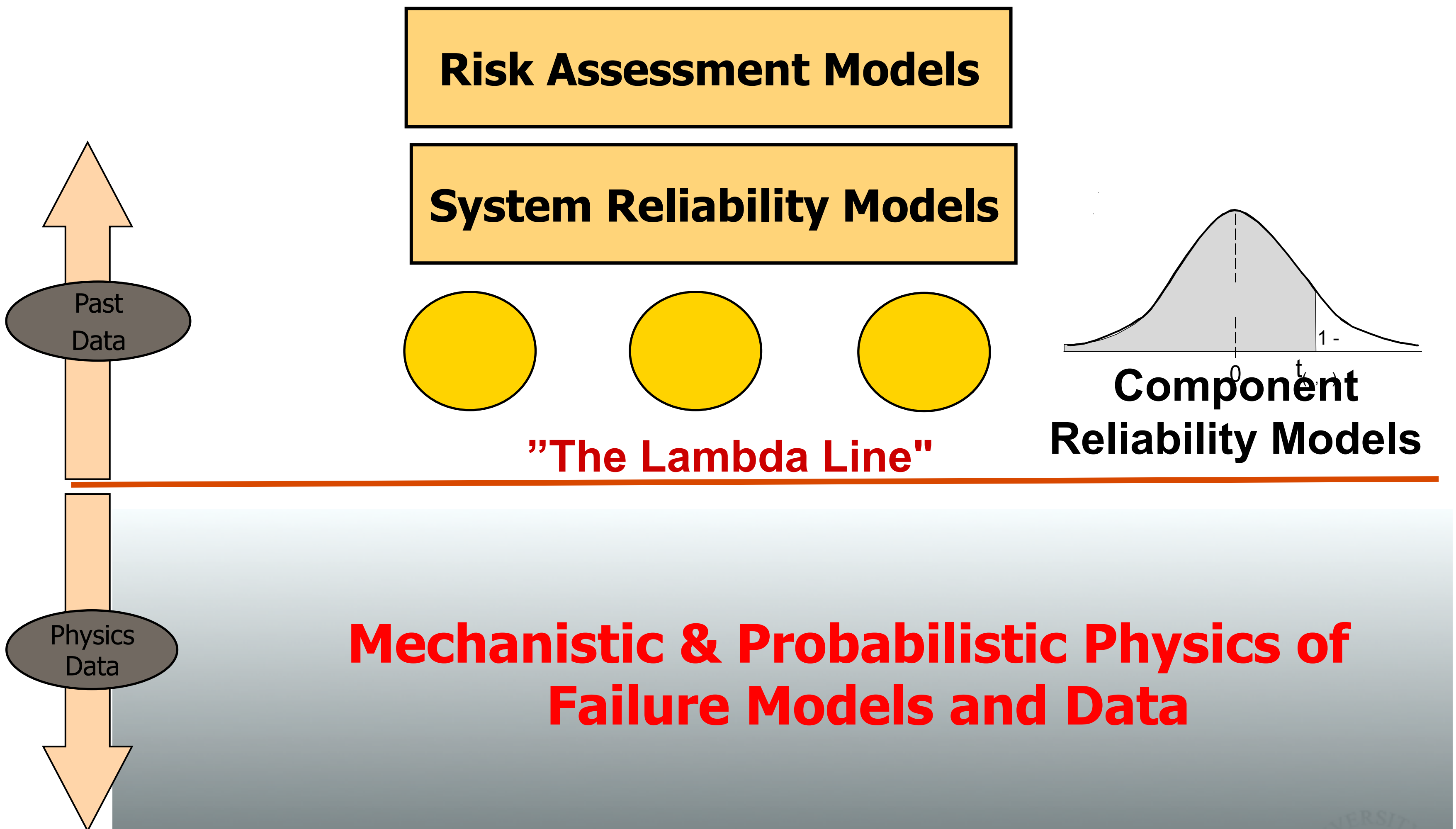
Cumulative Damage

$$\Lambda(t) = \int \lambda(u)\,du$$

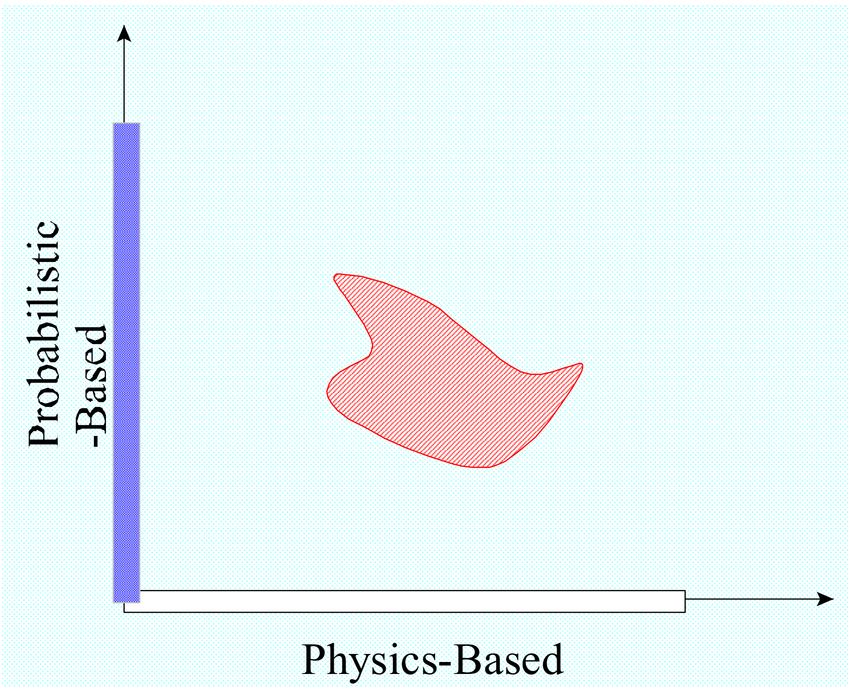
$$R(t_2 - t_1) = e^{-\int_{t_1}^{t_2} \lambda(t)\,dt}$$

$$\text{MTBF} = \int_0^{\infty} R(t - t_1)\,dt = \int_0^{\infty} e^{-\int_{t_1}^{t} \lambda(\tau)\,d\tau}\,dt$$

ROLE OF PHYSICS MODELS IN RELIABILITY



DIMENSIONS AND MODELING OF RELIABILITY



- Statistical / Probabilistic Models
- Empirical Physical Model
- Theoretical Physics Laws

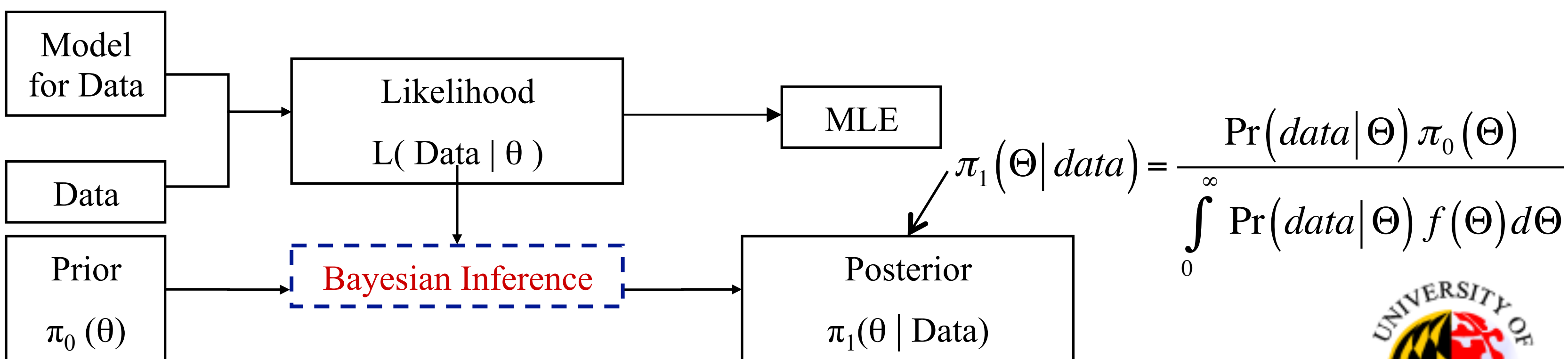
considerations

- Prediction of Future (Probability)
  - Future as copy of the past?
  - Uncertainties? (specially model uncertainties)
- Characterization of past performance (Statistics)
  - Classical
  - Bayesian

STATISTICAL AND PROBABILISTIC METHODS FOR ANALYZING DATA FOR MODEL DEVELOPMENT

- Probability Plotting
- Maximum Likelihood Estimation
- Bayesian Estimation

Type of Observation	Likelihood Function	Example Description
Exact Lifetimes	$L_i(\theta\|t_i) = f(t_i\|\theta)$	Failure time is known
Right Censored	$L_i(\theta\|t_i) = R(t_i\|\theta)$	Component survived to time t_i
Left Censored	$L_i(\theta\|t_i) = F(t_i\|\theta)$	Component failed before time t_i



WHY PoF-BASED MODELING?
TOWARD A SCIENCE-BASED RELIABILITY APPROACH

- Avoid relying solely on long and costly life tests and field data
  - Reduce development time / fast release of the design
  - Cost reduction
- Difficult to build several identical units for testing
  - Large systems like buildings, space vehicles
  - One-of-a-kind or highly expensive units
  - The products that must work properly at the first time
- No prototype to test during design
- Highly reliable units hard to break
  - Long life time
  - Internal control or safety related devices limit the stress
  - Higher stresses introduce other failure mechanisms
- Optimization purposes
- Predicting the occurrence of rare or extreme events

DEFINITION OF PHYSICS OF FAILURE MODELS

PoF is an engineering approach to reliability assessment that uses simulation of the physical models of degradation, damage and failure developed based on the science of failure mechanisms such as fatigue, fracture, wear, and corrosion through accelerated life testing and accelerated degradation testing

- **Benefits**
  - Less dependence on field failure data
  - Easily connected to other physical models
  - Address the underlying failure mechanisms
  - Reflect the physical experiences
  - More generic
  - Complex, but can model interacting mechanisms
- **Drawbacks**
  - Experience-based
  - Time consuming
  - Paradigm shift
  - Specific to one failure mode or mechanism
  - More expensive

TYPES AND FORMS OF PoF MODELS

TYPES

(Life) vs. (Stress) Model

$$t = f(s_o, s_e, g, m, d)$$

(Cumulative Degradation or Damage) vs. (Stress) Model

$$D(t) = f(t,\ s_o, s_e, g, m, d)$$

Where, S_o = Operational Stresses

S_e = Environmental Stresses

g = Geometry related factors

m = material properties

d = initial defects, flaws, etc.

FORMS

Linear: $y = ax + b$

Exponential: $y = b\, e^{ax}$

Power: $y = bx^a$

Logarithmic: $y = a \ln(x) + b$

Combination of these forms

Examples:

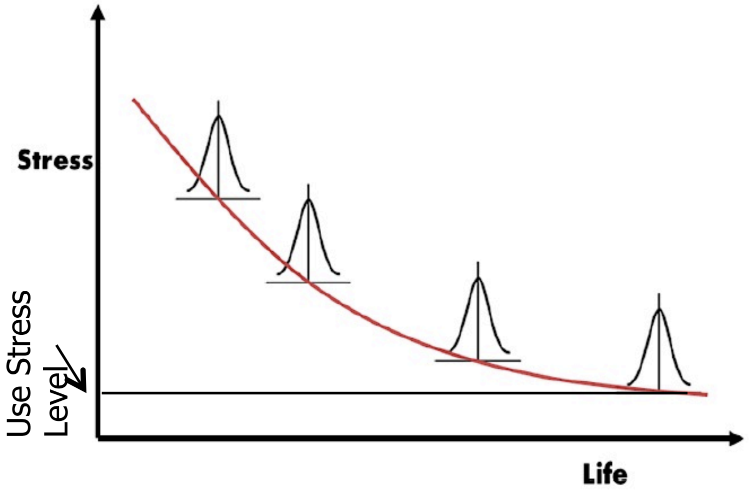
$$Life = Ae^{\frac{E_a}{KT}}$$

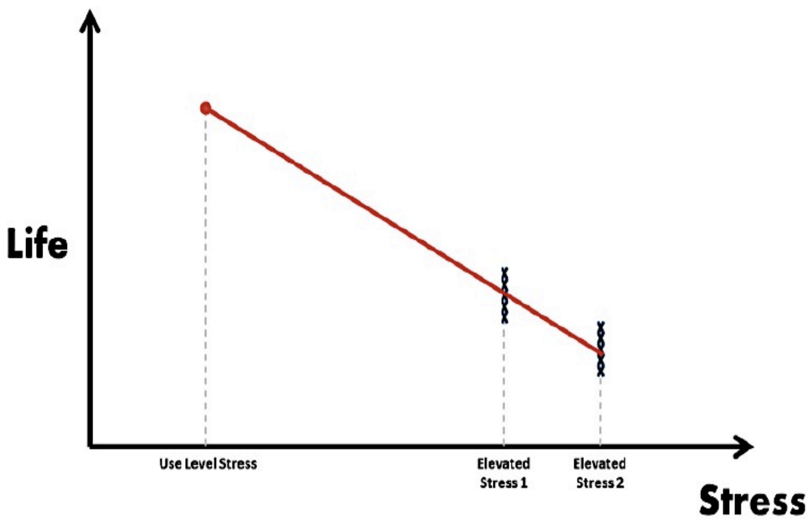
$$Life = \frac{1}{K(Stress)^n}$$

$$Life = \frac{1}{Stress} e^{-\left(A - \frac{B}{Stress}\right)}$$

LIFE-STRESS AND STRESS-LIFE MODELS IN ALT

- L-S model relates a given percentile of life to applied stresses or failure causing agents
- It is a mathematical model developed per failure mechanism and per failure mode



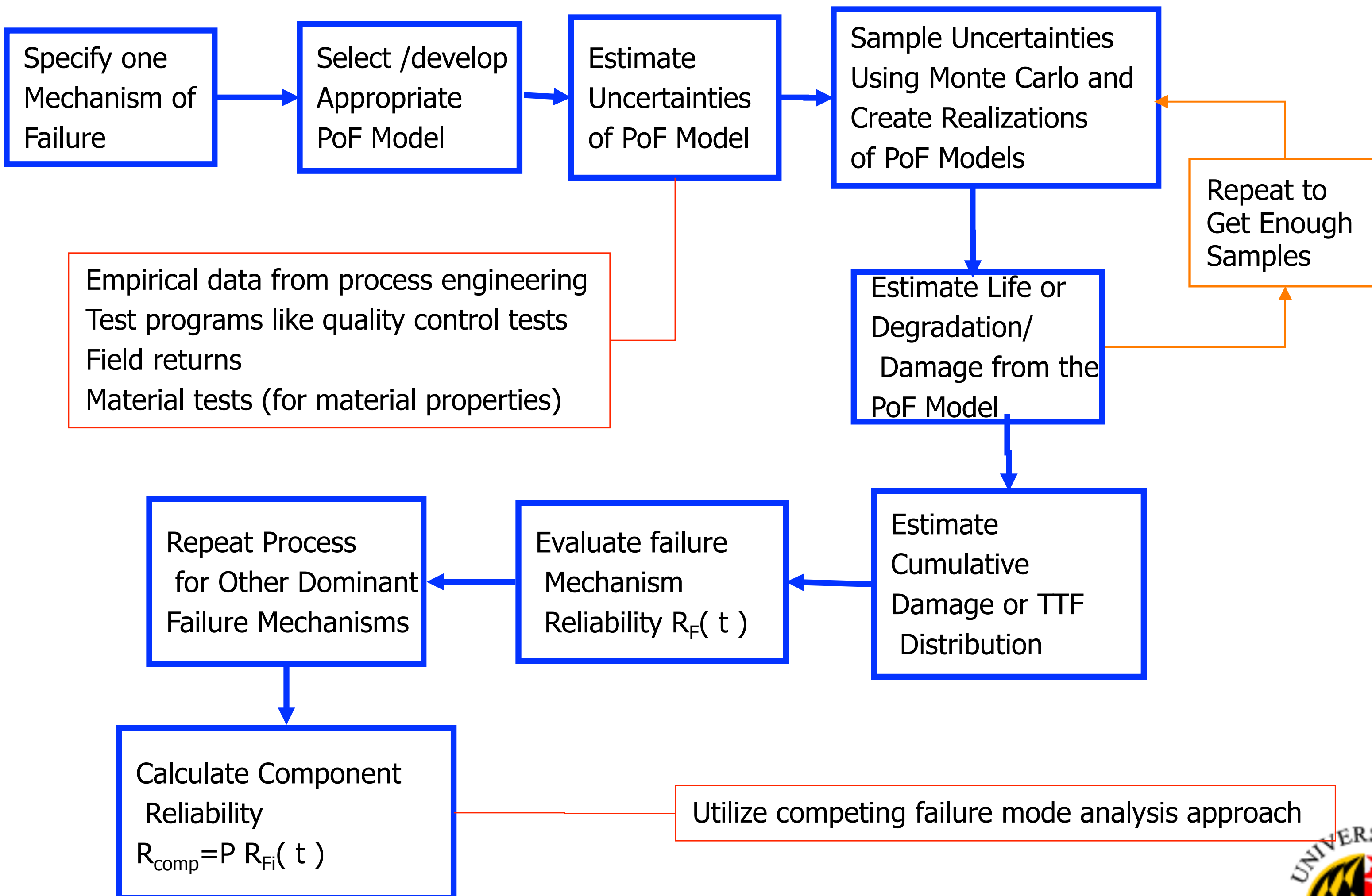


- ALT is about gathering life data and describing them in form of a probability density function (PDF) of life (time-to-failure) as shown in above figures

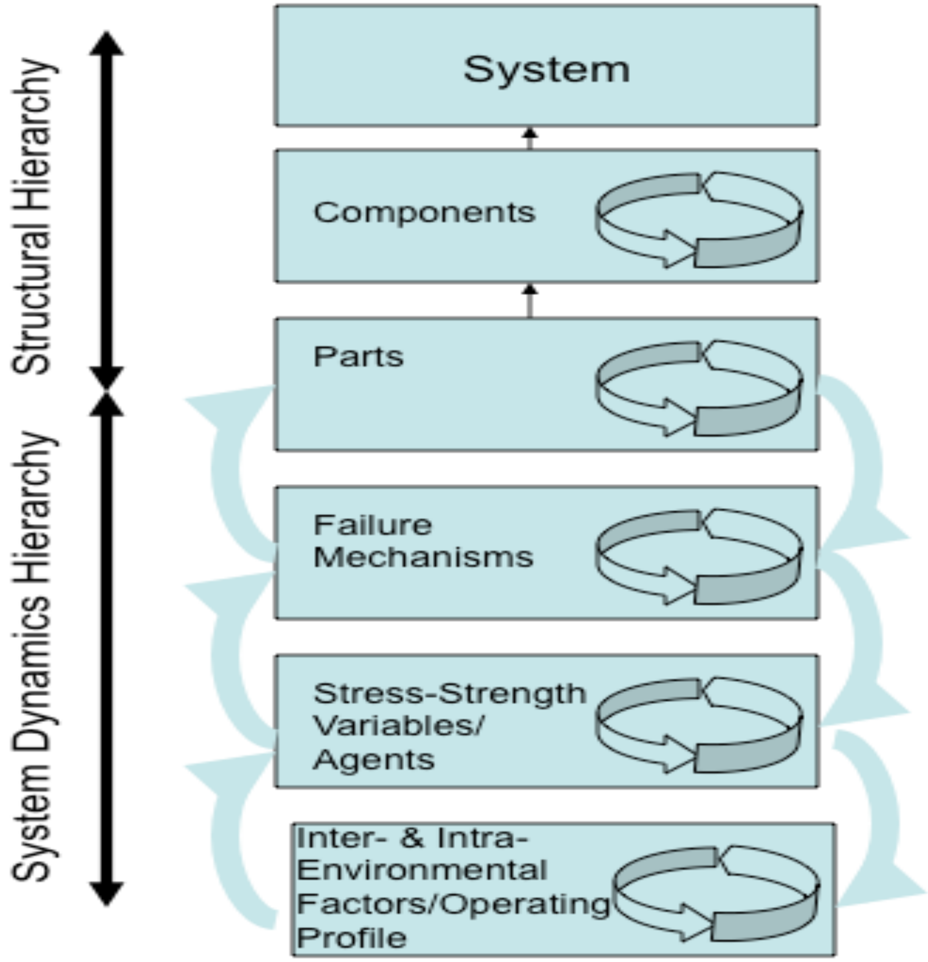
EXAMPLES OF FAILURE MECHANISMS RELATED TO MECHANICAL UNITS AND THEIR AGENTS/STRESSES

Mechanism	Material	Accelerated stress	Measured properties
Fatigue	Metals, plastics, glass, ceramics, composites	Load, temperature, chemicals (water, hydrogen, oxygen)	Residual life, residual strength, cumulative damage (CD)
Corrosion/oxidation	Metals, food & drugs	Concentration of the chemicals, activators, temperature, voltage, mechanical load (stress-corrosion)	Probabilistic degradation model based on physical mechanism
Creep	Metals, plastics	Temperature and mechanical load, load cycling, chemical contaminants (e.g., water, hydrogen, fluorine)	Plastic deformation under constant load
Cracking	Metals, plastics, glass, ceramics, composites	Mechanical stress, temperature, chemicals (humidity, hydrogen, alkalis, acids)	Degradation testing for stability & shelf life
Wear	Rubber, polymers, metals	Speed, load (magnitude, type), temperature, lubrication, chemicals (humidity)	Degradation testing of mechanical properties

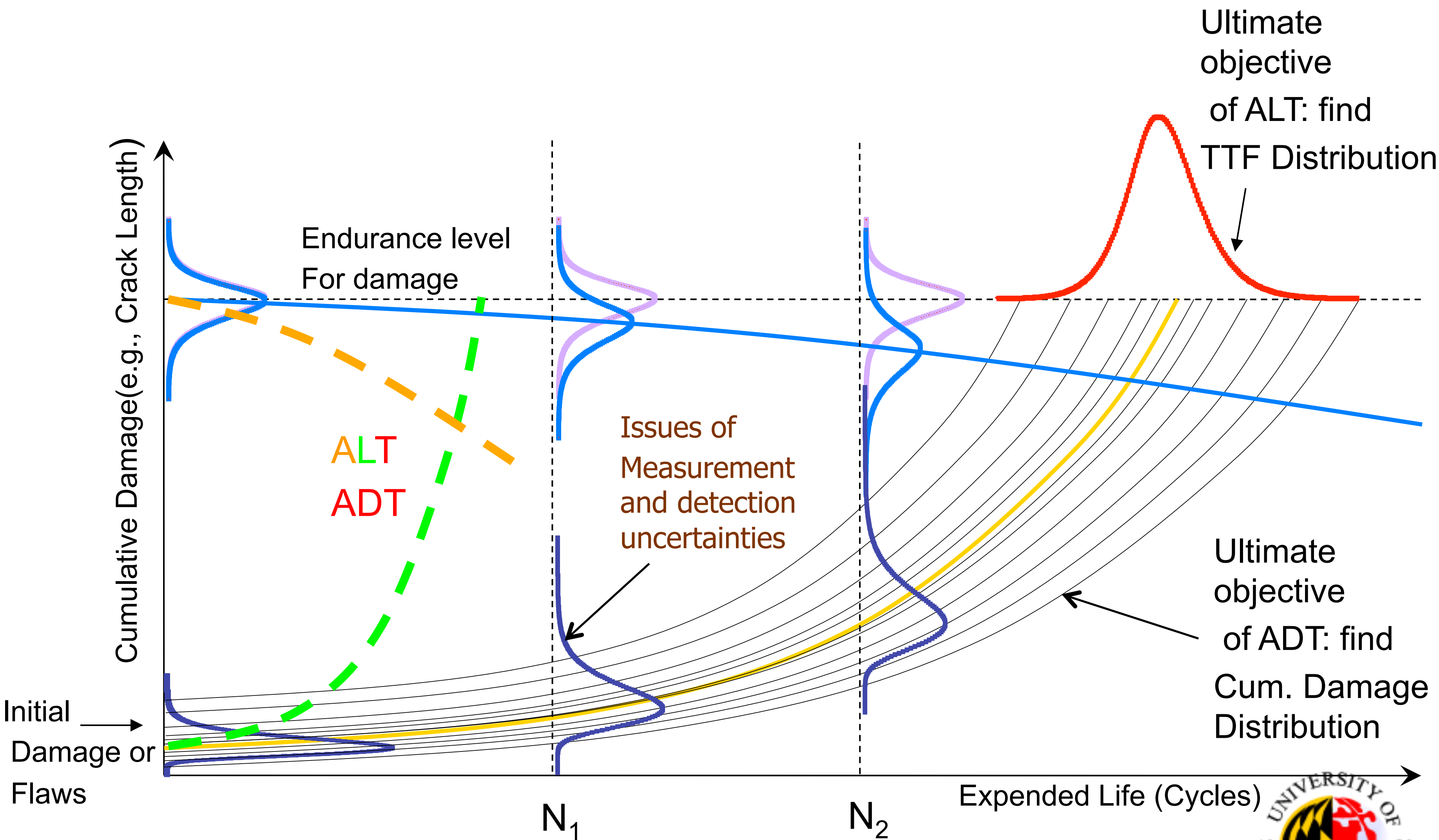
PROBABILISTIC PoF (PPoF) APPROACH



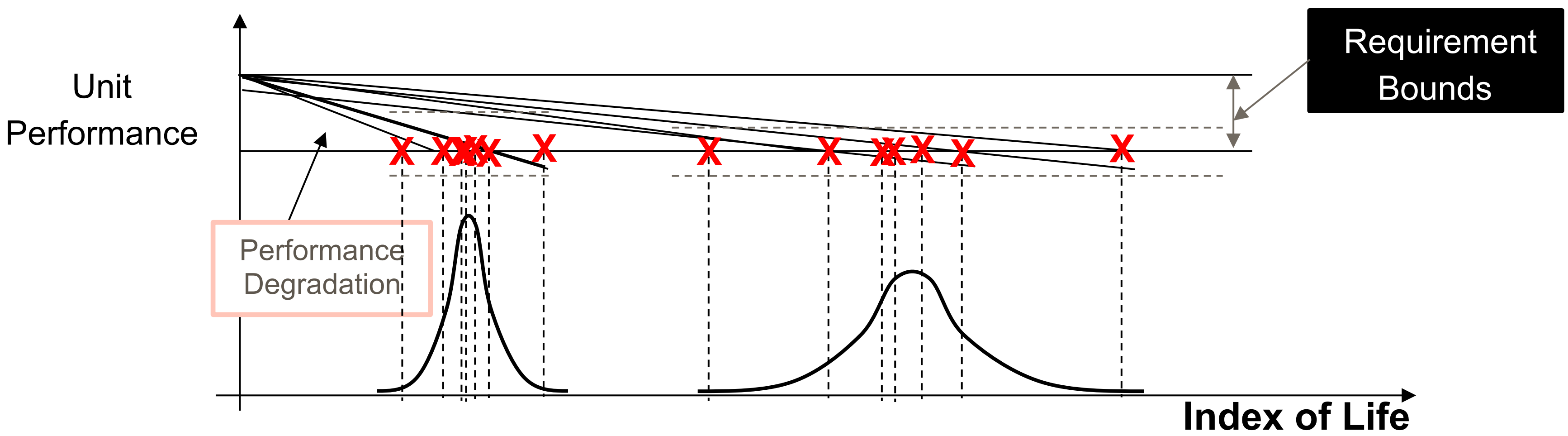
SYSTEM HIERARCHY CONSIDERED IN PoF ANALYSIS



DEPICTION OF A DAMAGE-ENDURANCE MODEL



ALTERNATIVE BUT SIMILAR MODELS: PERFORMANCE-REQUIREMENT MODEL



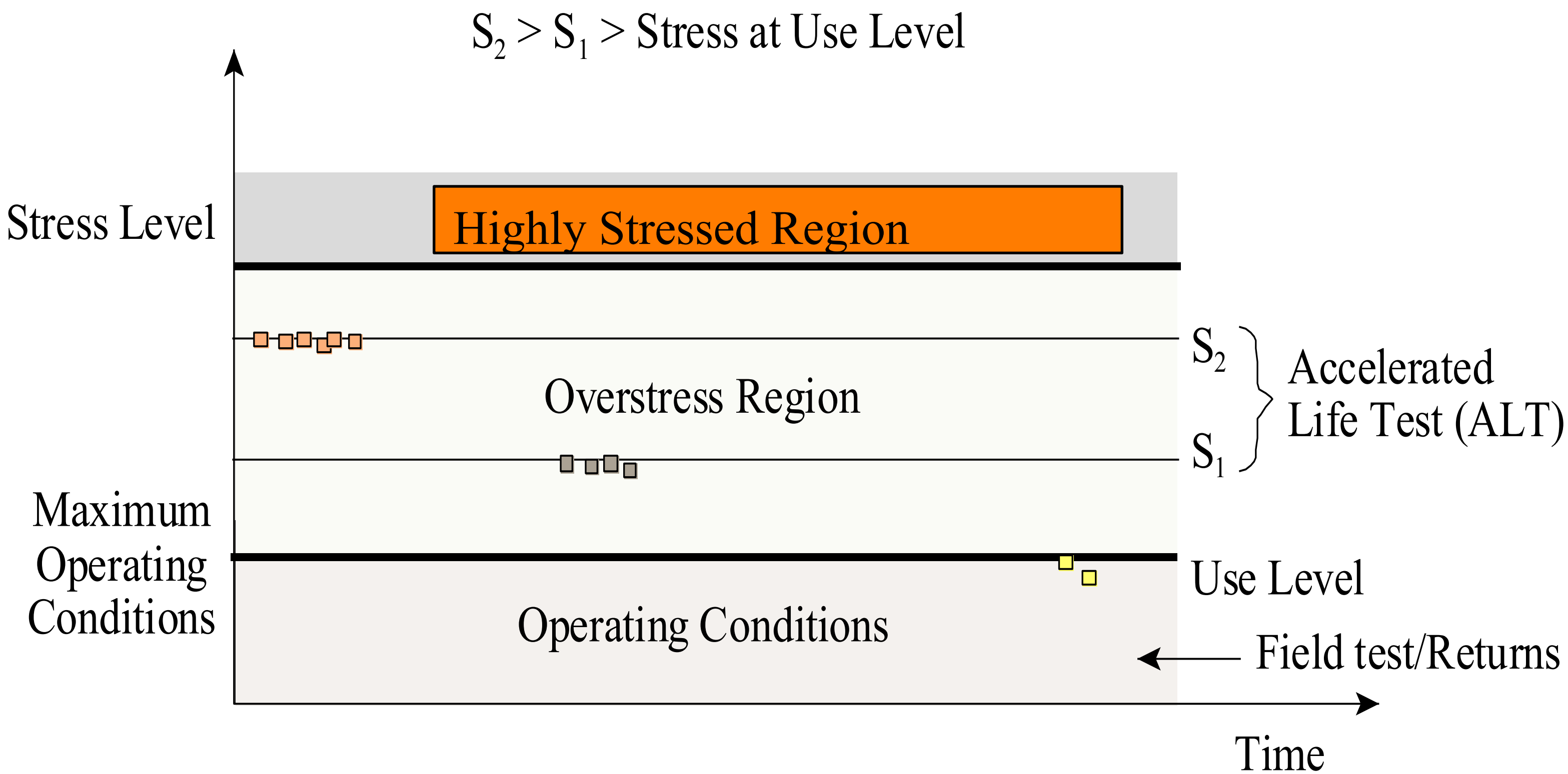
Examples:

- Degradation of safety margin
- Degradation of efficiency

Assumption:

- Aging and operational changes lead to degradation of performance and safety margin

ACCELERATED LIFE TEST STRESS LEVELS



ALT Provides more failures within shorter test durations

EXAMPLE OF ALT / ADT MATHEMATICS: EMPIRICAL APPROACH

- Consider Lognormal as Distribution of Time to Failure (or Cumulative Damage)

$$f(t\ or\ D) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{[\ln(t)-\ln(\mu)]^2}{2\sigma^2}}$$

$\mu =$ Median

$\sigma =$ Standard Deviation

$t =$ Time to Failure or Cumulative Damage

- Inverse Power Law as acceleration life-stress model

$$\mu = \frac{1}{KS^n}$$

$\mu =$ Median

$K, n =$ Constants to be determined from data analysis

$S =$ Stress

- Final Joint Distribution of Life-Stress Model

$$f(t \mid S, K, n, \sigma) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{[\ln(t)+\ln(K)+n\ln(S)]^2}{2\sigma^2}}$$

Data Analysis and Estimation Methods:

- Plotting
- MLE
- Bayesian

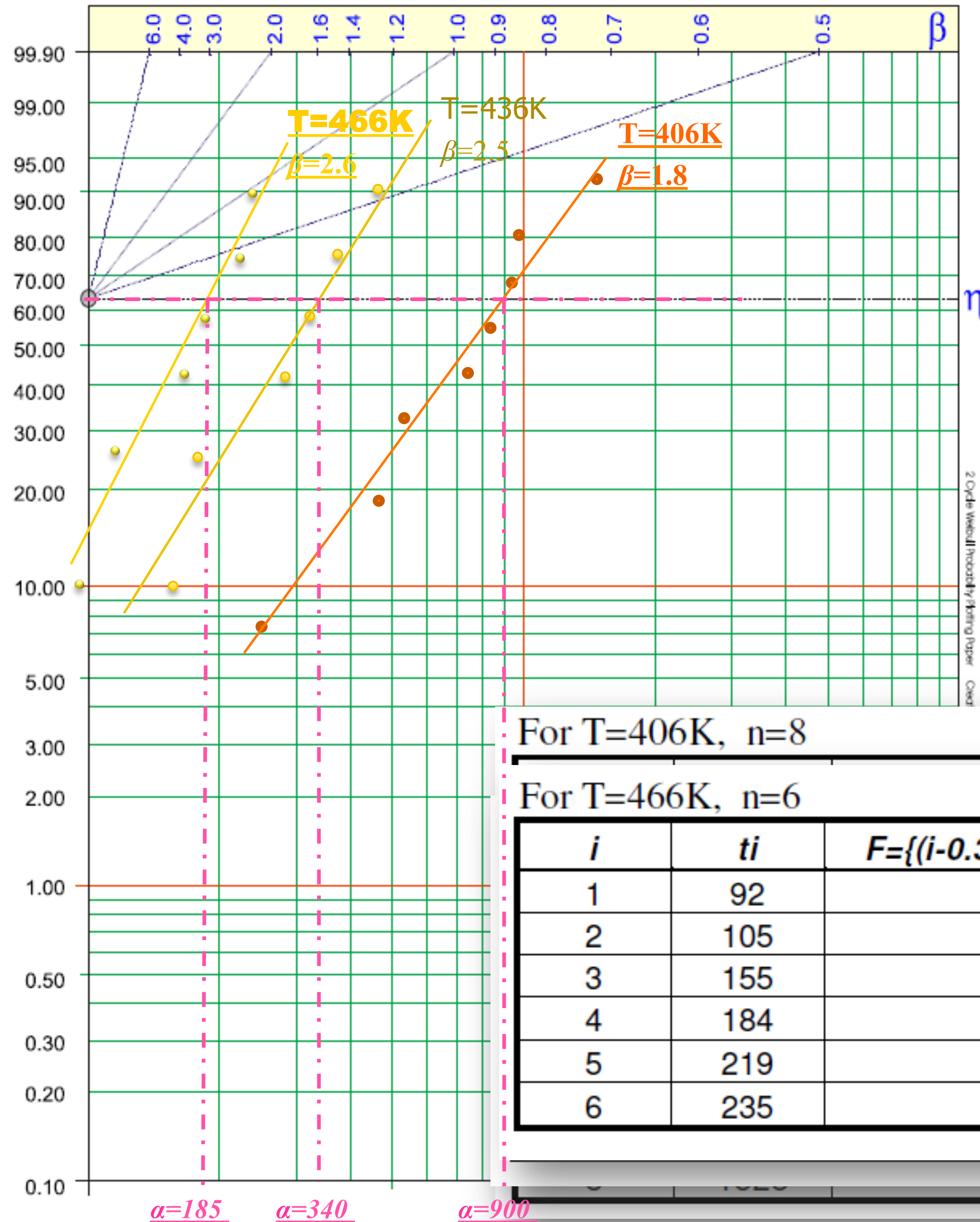
Step 1:

Plotting The Life Distribution @ each stress level

Step 2:

Finding the parameter of the Distribution:

Here β and α



For T=406K, n=8

For T=466K, n=6

i	ti	F={(i-0.375)/(n+0.25)}*100
1	92	10.00
2	105	26.00
3	155	42.00
4	184	58.00
5	219	74.00
6	235	90.00

ALT PLOTTING EXAMPLE (Cont.)

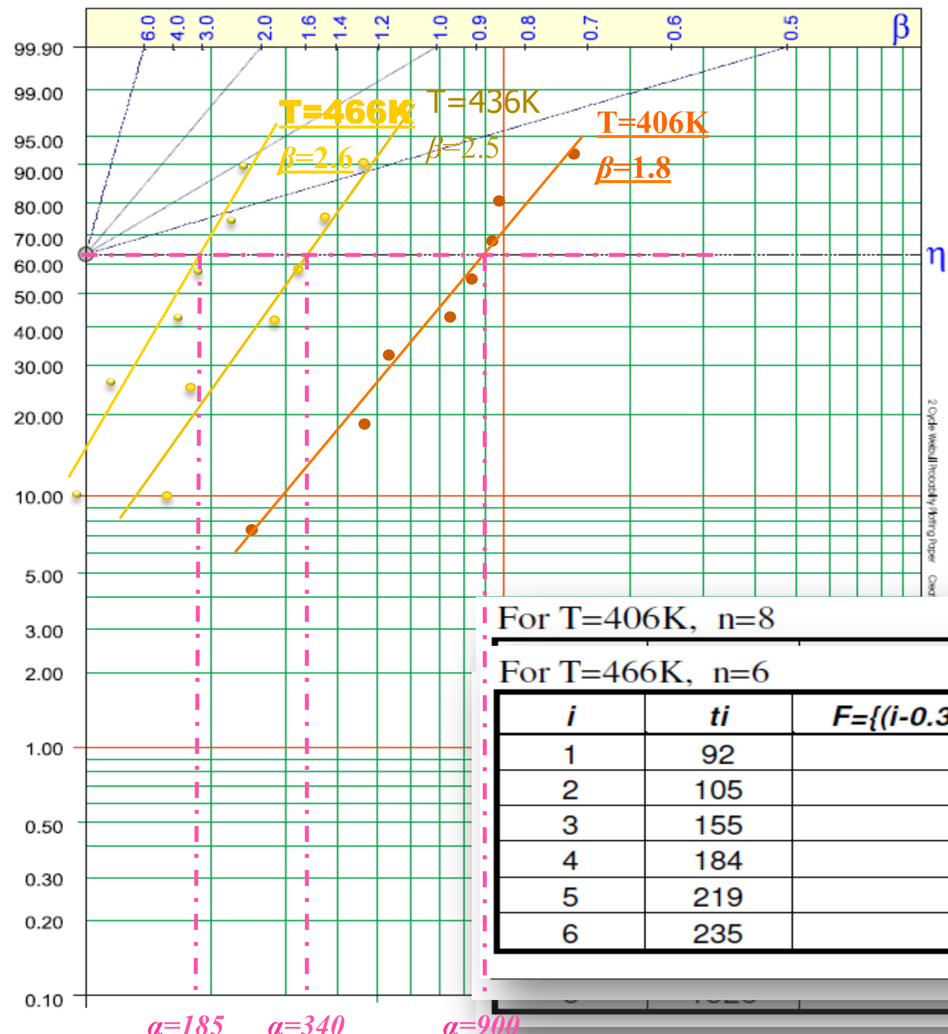
Step 1:

Plotting The Life Distribution @ each stress level

Step 2:

Finding the parameter of the Distribution:

Here β and α



For T=406K, n=8

For T=466K, n=6

i	*ti*	*F={(i-0.375)/(n+0.25)}*100*
1	92	10.00
2	105	26.00
3	155	42.00
4	184	58.00
5	219	74.00
6	235	90.00

ALT PLOTTING EXAMPLE (Cont.)

- Results:
  - For T=406°K, β_1=1.8 and α_1=900 (i.e., $t_{63.2\%}$ of life at this stress)
  - For T=436°K, β_1=2.5 and α_1=340 (i.e., $t_{63.2\%}$ of life at this stress)
  - For T=466°K, β_1=2.6 and α_1=185 (i.e., $t_{63.2\%}$ of life at this stress)
- Constant shape parameter (averaged values) β =(1.8+2.5+2.6)/3=2.3
- Stress-Life mode $t_{63.2\%} = A_{63.2\%} e^{\frac{E_{a63.2\%}}{KT}}$ OR $Ln(t_{63.2\%}) = L(A_{63.2\%}) + \frac{E_{a_{63.2\%}}}{KT}$
- Or simply Y=mX+b, where Y=$t_{63.2\%}$; m= $\frac{E_{a_{63.2\%}}}{K}$; and X=1/T
- Find the least square fits to three Y_i vs. X_i and find best m and b (and the corresponding E_a/K and A.
- Final Results: E_a/K=4680.35, A=0.0106
- Estimation of Mean 63.2% life for a 50 °C "use stress" =

$$t_{63.2\%} = A_{63.2\%} e^{\frac{E_{a63.2\%}}{KT}} = 0.0106 e^{\frac{4680.35}{273+50}} = 20,813.3 \quad hr$$

$$AF_{63.2\%}^{Use\,vs.\,Highest\,stress} = \frac{20813.3}{185} = 112.5$$

THEORETICAL ALT /ADT MODELS: THERMODYNAMICS AND INFORMATION THEORY APPROACH

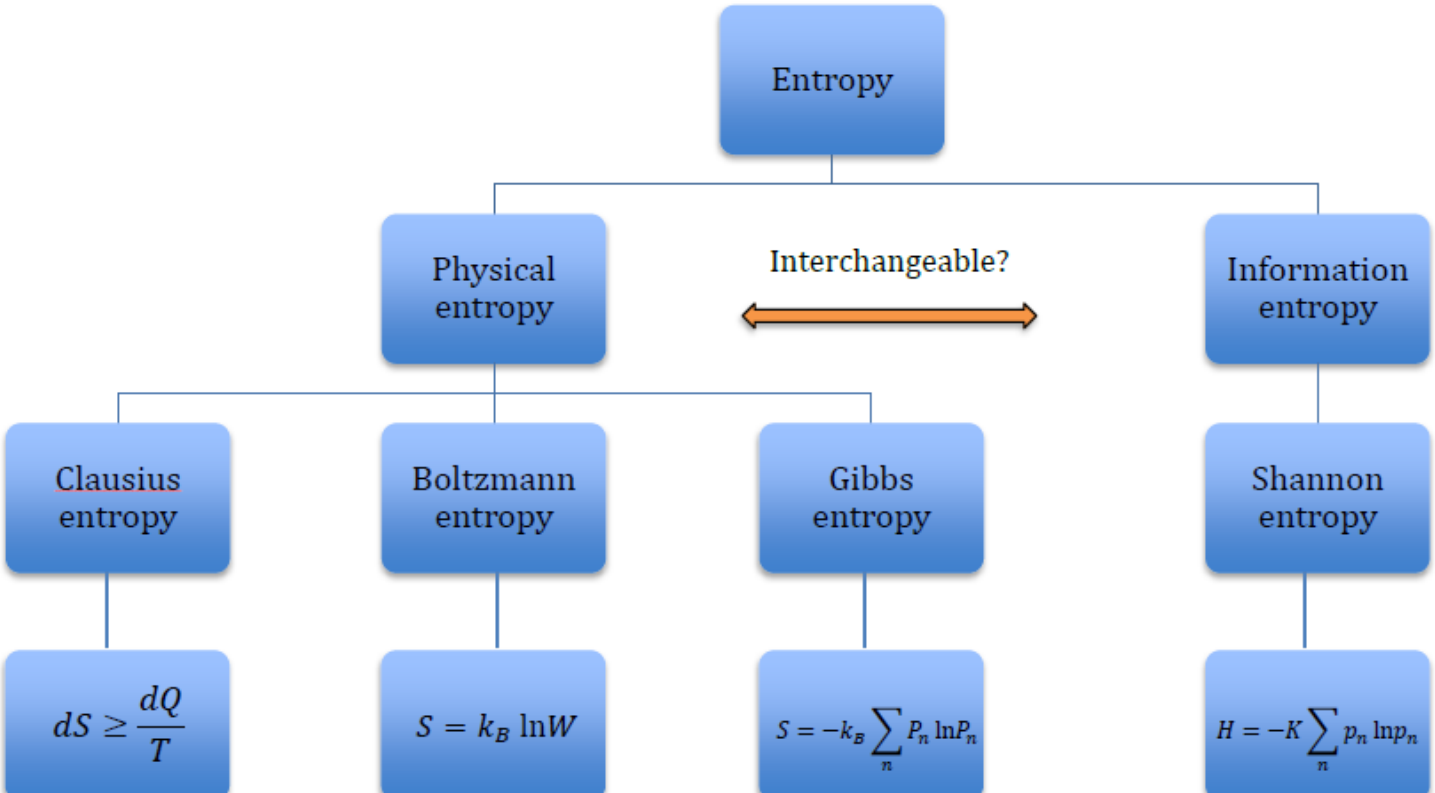
TOWARD A FULLY SCIENCE-BASED APPROACH

- Degradation and ultimate failure may be thought of as dissipative processes within materials of components and systems
- Thermodynamics offers a universal description of all natural phenomena (e.g., chemical reactions, permanent deformation of solids, friction, release of heat, etc.) that underlie failure mechanisms
- Thermodynamic laws must be obeyed at all length from micro-scale, meso-scale to macro-scale
- Constitutive damage models can be derived from thermodynamic laws and considerations
- Possible generalization of entropy based damage would bridge the gap across all scales

THEORETICAL ALT /ADT MODELS: THERMODYNAMICS AND INFORMATION THEORYAPPROACH (CONT.)

-2nd LAW OF THERMODYNAMICS

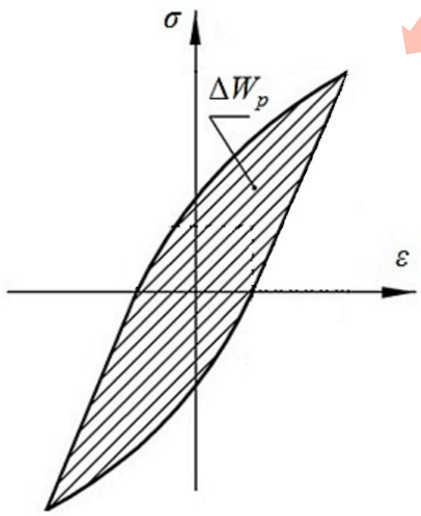
-INFORMATION THEORY



THERMODYNAMIC ALT /ADT MODELS

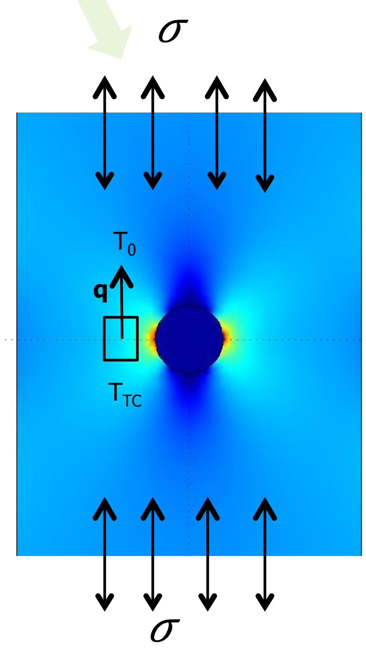
Accumulated entropy generation, s_i up to crack initiation time, t_i :

$$s_i = \int \left(\frac{1}{T}\boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}_p - \frac{1}{T}A_k\dot{V}_k - \frac{1}{T}Y\dot{D} - \frac{1}{T^2}\boldsymbol{q}.\,\boldsymbol{grad}T \right) dt$$



Hysteresis Loop

σ : stress
q: heat flux
T_0: Ambient Temperature
T_{TC}: thermocouple temperature



MODELING DAMAGE VIA QUASI-NONLINEAR THERMODYNAMICS

From irreversible thermodynamics, entropy generation can be defined in terms of thermodynamic force, *X*, and thermodynamic flux, *J*, as:

$$\frac{d_i S}{dt} = \sum_i X_i J_i$$

Degradation mechanism	Entropy generation	Damage
(X_1, J_1) (X_2, J_2) $\vdots$ (X_n, J_n)	$X_1 J_1$ $X_2 J_2$ $\vdots$ $X_n J_n$	$D_1 = D_1(X_i, J_i)$ $D_2 = D_2(X_i, J_i)$ $\vdots$ $D_n = D_n(X_i, J_i)$
Total effect	$\frac{d_i S}{dt} = \sum_{i=1}^{n} X_i J_i$	$D = \sum_{i=1}^{n} D_i$

Each thermodynamic flux, J_i, depends not only on its conjugate thermodynamic force X_i, but also on all other forces X_j ($i \neq j$):

$$J_i = J_i(X_1, \ldots, X_n) \qquad (i = 1, \ldots, n)$$

De Groot & Mazur, Non-equilibrium thermodynamics. Amsterdam: North-Holland Pub. Co. 1962

MODELING DAMAGE VIA QUASI-NONLINEAR THERMODYNAMICS (CONT.)

For a large class of irreversible phenomena and under a wide range of experimental conditions, the irreversible fluxes are linear function of the thermodynamic forces:

$$J_i = L_{ik} X_k \quad (i,\, k = 1, \ldots, n)$$

L_{ij} are phenomenological coefficients which can be used to quantify coupling (synergistic) effect. Onsager reciprocal relations for linear phenomena is:

$$L_{ij} = L_{ji} \quad \forall i \neq j$$

For example: suppose subscript *c* denotes corrosion and *f* denotes fatigue, entropy production in the form of forces and fluxes is given by:

$$\frac{d_i S}{dt} = X_c J_c + X_f J_f$$

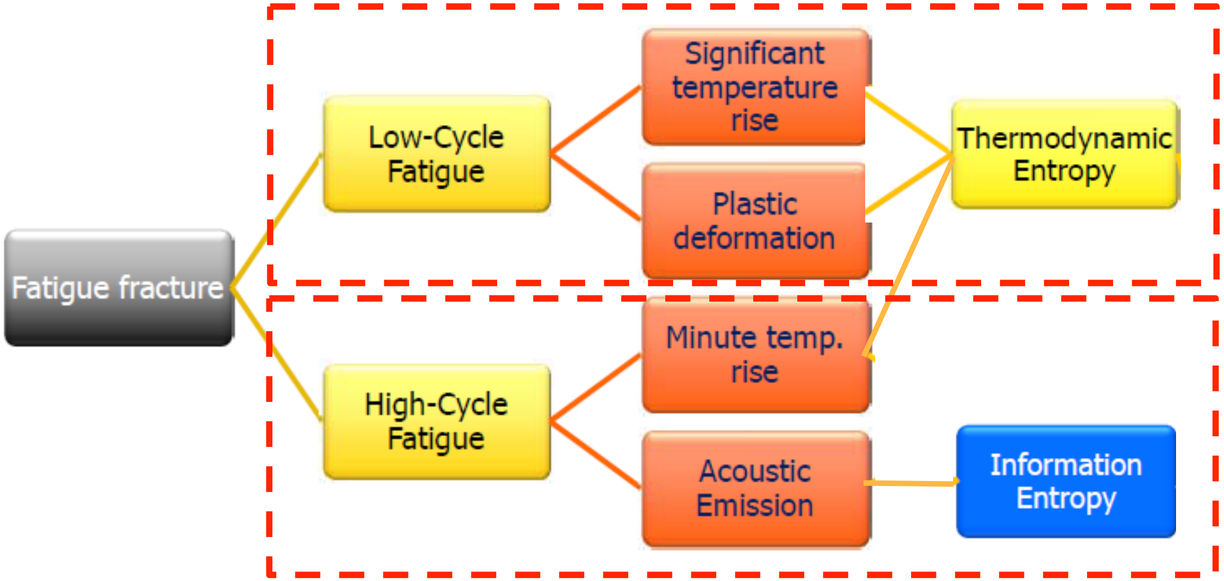
where the synergistic effect is manifested in coefficients *Lcc*, *Lff*, *Lcf* and *Lfc*:

$$\begin{bmatrix} J_c \\ J_f \end{bmatrix} = \begin{bmatrix} L_{cc} & L_{cf} \\ L_{fc} & L_{ff} \end{bmatrix} \begin{bmatrix} X_c \\ X_f \end{bmatrix}$$

$$\frac{dD}{dt} = \sum_j B X_j J_j \;\rightarrow\; H(t) \propto D(t) \;\rightarrow\; R(t) = e^{-H(t)}$$

De Groot & Mazur, Non-equilibrium thermodynamics. Amsterdam: North-Holland Pub. Co. 1962

EXAMPLES OF APPLICATIONS IN ADT AND PHM



Work initiated by Whaley et al., (1983) followed by several researchers, Basaran et al., (1998-2007), Tucker et al., (2012), Chan et al., (2012) and LSU

Entropy in the information context has a rich history of research in damage detection and structural health monitoring. UMD attempts to answer the question: *what is the advantage of a signal's entropy over the conventional AE hit method for structural health monitoring?*

Work initiated by Klamecki (1980) followed by several researchers, Doelling et al., (2000), Bryant (2009), and LSU

CONCLUSIONS

- ADT and ALT as Alternative to Traditional Reliability and Risk Assessment
- Methods are practical and provide component-specific life assessment
- Inventory of PoF empirical models exists
- Future research aims to connects fundamental science to reliability assessment