Assessing Reliability Using Developmental and Operational Test Data

Martin Wayne, PhD, U.S. AMSAA
Mohammad Modarres, PhD, University of Maryland, College Park

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Agenda

• Background
• Overarching Model Framework
• Reliability growth in Developmental Testing (DT)
• Combing DT and Operational Testing (OT)
Results
• Performance Comparisons
• Conclusions and Future Work
Background

- New military product development generally contains developmental reliability growth testing as design matures to final state

- Developmental testing environment may not completely represent operational usage environment
  - Operators are different
  - Loads and stresses may be different

- Reliability currently assessed in single operational test
  - Final mature configuration
  - Generally short and expensive tests
Problems with Current Operational Assessment

- Short test lengths can lead to “flat” Operating Characteristic (OC) curves. This often results in test plans in which no failures or a single failure are allowable.
- Resource constraints and technology maturity may make demonstration infeasible.

\[ OC(M) = \sum_{i=0}^{c} \left( \frac{T}{M} \right)^i \exp \left[ -\frac{T}{M} \right] \]

M = true but unknown reliability of the system and c = maximum number of allowable failures T = total demonstration test length
Proposed Alternative Assessment

• Utilize data available from developmental testing within Bayesian framework
  – Account for reliability growth during development
  – Account for differences in test environment/conduct

• Benefits include:
  – Narrower probability intervals
  – Reduced testing requirements, lower costs, etc.
  – Can use additional data sources in reliability assessment
Overarching Framework

Developmental Test (DT) Data

Failure Mode Posterior Result

System Level Posterior DT Distribution

Updated Prior Distribution

Operational Test (OT) Data

Posterior Distribution

Results from DT form prior distribution for OT

System Level Posterior OT Result
Likelihood for DT Reliability Growth

• Accounts for arbitrary corrective action strategy

• For each failure mode, $i$, in the system assume:
  – Failure intensity is constant before and after corrective action
  – $n$ failures in demonstration test time $T_{DT}$ with $n_1$ occurring before corrective action
  – Corrective action at time $v$ with Fix Effectiveness Factor (FEF) $d$

• Likelihood given by

$$l(t_{i,1}, t_{i,2}, \ldots, t_{i,n_i}, n_i, n_{i,1} | \lambda_i) = (1-d_i)^{n_i-n_{i,1}} \lambda_i^{n_i} \exp\left(-\lambda_i \left[ v_i + (1-d_i)(T_{DT} - v_i) \right]\right)$$

Likelihood allows for arbitrary reliability growth
Choice of Prior Distribution on Failure Intensity

- Assumes failure mode failure intensities realized from common Gamma($\alpha$, $\beta$) distribution
- Gamma Follows “vital-few, trivial-many” construct
- Can use Empirical Bayes to estimate parameters

\[
p(\lambda) = \frac{\lambda^{\alpha-1}}{\Gamma(\alpha)\beta^\alpha} \exp\left( -\frac{1}{\beta} \lambda \right)
\]

Provides inherent connection between failure modes

\[
p(\lambda_i | n_i) = \frac{\lambda_i^{\alpha+n_i-1}}{\Gamma(\alpha+n_i)} \left[ \frac{1}{\beta} + v_i + (1-d_i)(T_{DT} - v_i) \right]^{-(\alpha+n_i)} \exp \left[ -\lambda_i \left( \frac{1}{\beta} + v_i + (1-d_i)(T_{DT} - v_i) \right) \right]
\]
System Level Result

- System level estimate:
  - Summing over $m$ total failure modes
  - Take limit as $m$ becomes large
  - $n$ number of failures for each of the $m$ observed failure modes, i.
- For $m$ observed modes, system level mean is

\[
E[\lambda_s] = \sum_{i=1}^{m} \left( \frac{(1-d_i)n_i}{\frac{1}{\beta} + v_i + (1-d_i)(T-v_i)} \right) + \left( \frac{\lambda_B}{1 + \beta T} \right)
\]

where $\lambda_B = m\alpha\beta \equiv$ prior mean

Estimate includes contribution from unobserved modes
System Level Posterior Distribution

- Posterior can be simulated to determine approximate distribution
  - Exactly Gamma if corrective actions are delayed
  - Gamma approximation reasonable for arbitrary corrective action strategies

\[ \mu = E[\lambda_s] \]
\[ \beta' = \frac{Var[\lambda_s]}{E[\lambda_s]} \]

\[ \lambda_s \sim \text{Gamma}\left(\alpha' = \frac{\mu}{\beta'}, \beta'\right) \]

Can use mean and variance to develop system level posterior
Incorporating Operational Data

• Generally have increased failure intensity in operational environment
  – DT conditions more benign, human factors, etc.

• Define MTBF as reciprocal of mode failure intensity $\lambda$

• Assume 100$\gamma$% decrease (degradation factor) in instantaneous MTBF ($1/\lambda$) such that

\[
\lambda_{DT} = (1 - \gamma)\lambda_{OT}
\]

• Transformed prior found using properties of Gamma distribution

\[
\lambda_{OT} | \gamma = \frac{\lambda_{DT}}{(1 - \gamma)} \sim Gamma \left[ \alpha', \frac{\beta'}{(1 - \gamma)} \right]
\]

Scaled prior accounts for reliability degradation
Marginal Posterior Distribution

- Assume $n_2$ failures in operational test length $T_2$
- Treats degradation factor $\gamma$ as nuisance parameter
- Marginal posterior development
  - Compute joint posterior
  - Compute marginal distribution by integrating over nuisance parameter

\[
l (t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} | \lambda_{OT}) = \lambda_{OT}^{n_{OT}} \exp(-\lambda_{OT} T_{OT})
\]

\[
p (\lambda_{OT} | n_{OT}) = \int_{\gamma} \int_{\Lambda, \Gamma} \frac{p (\gamma) p (\lambda_{OT} | \gamma) l (t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} | \lambda_{OT})}{p (\gamma) p (\lambda_{OT} | \gamma) l (t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} | \lambda_{OT})} \, \partial \lambda_{OT} \, \partial \gamma
\]

- Use Beta prior distribution for $\gamma$

Marginal posterior probabilistically accounts for degradation
OT Posterior Assessment

- Posterior mean is scaled mean for Gamma distribution

\[
E[\lambda_s \mid n] = \left( \frac{\mu}{\beta'} + n_2 \right) \left( \frac{1}{\beta' + T_2} \right)
\]

\[
= \frac{2_F_1 \left[ \frac{\mu}{\beta'} + n_2 + 1, a, a + b + \frac{\mu}{\beta'}, \frac{1}{\beta' + T_2} \right]}{2_F_1 \left[ \frac{\mu}{\beta'} + n_2, a, a + b + \frac{\mu}{\beta'}, \frac{1}{\beta' + T_2} \right]}
\]

where \(\mu, \beta'\) defined as on slide 9

Standard posterior mean for Gamma

Ratio of Hypergeometric functions accounts for DT/OT degradation
System Level OT Posterior Distribution

• Exact system-level posterior can be simulated using Markov Chain Monte Carlo methods
• Posterior well approximated with Gamma distribution
• Use approximate Gamma to develop probability intervals

Can use mean and variance to develop system level posterior
Performance Comparisons

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial DT MTBF</th>
<th>DT Length</th>
<th>OT Length</th>
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<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>1000</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>2000</td>
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</tr>
<tr>
<td>3</td>
<td>100</td>
<td>2000</td>
<td>500</td>
</tr>
</tbody>
</table>

Simulation developed to examine relative error between model estimate and “true” value.

Bayesian approach performs better than current methods.

Full error distribution comparisons available in paper.
Conclusions/Future Work

• Use of reliability data from developmental testing provides additional information that increases performance of overall reliability estimate

• Bayesian probabilistic approach provides flexibility
  – Can utilize multiple information sources
  – Can include additional sources of uncertainty

• Current/future efforts include:
  – Modeling uncertainty on FEF values
  – Developing prior information from additional data sources
  – Analogous results for discrete systems