Assessing Reliability Using Developmental and Operational Test Data

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Agenda

• Background
• Overarching Model Framework
• Reliability growth in Developmental Testing (DT)
• Combing DT and Operational Testing (OT)
  Results
• Performance Comparisons
• Conclusions and Future Work
Background

- New military product development generally contains developmental reliability growth testing as design matures to final state

- Developmental testing environment may not completely represent operational usage environment
  - Operators are different
  - Loads and stresses may be different

- Reliability currently assessed in single operational test
  - Final mature configuration
  - Generally short and expensive tests
Problems with Current Operational Assessment

• Short test lengths can lead to “flat” Operating Characteristic (OC) curves. This often results in test plans in which no failures or a single failure are allowable.

• Resource constraints and technology maturity may make demonstration infeasible.

\[ OC(M) = \sum_{i=0}^{c} \left( \frac{T}{M} \right)^i \exp\left( -\frac{T}{M} \right) \]

M = true but unknown reliability of the system and
c = maximum number of allowable failures
T = total demonstration test length
Proposed Alternative Assessment

• Utilize data available from developmental testing within Bayesian framework
  – Account for reliability growth during development
  – Account for differences in test environment/conduct

• Benefits include:
  – Narrower probability intervals
  – Reduced testing requirements, lower costs, etc.
  – Can use additional data sources in reliability assessment
Overarching Framework

Results from DT form prior distribution for OT
Likelihood for DT Reliability Growth

• Accounts for arbitrary corrective action strategy
• For each failure mode, i, in the system assume:
  – Failure intensity is constant before and after corrective action
  – \( n \) failures in demonstration test time \( T_{DT} \) with \( n_1 \) occurring before corrective action
  – Corrective action at time \( v \) with Fix Effectiveness Factor (FEF) \( d \)
• Likelihood given by

\[
l(t_{i,1}, t_{i,2}, \ldots, t_{i,n_i}, n_i, n_{i,1} | \lambda_i) = (1 - d_i)^{n_i - n_{i,1}} \lambda_i^{n_i} \exp\left(-\lambda_i \left[ v_i + (1 - d_i)(T_{DT} - v_i) \right]\right)
\]

Likelihood allows for arbitrary reliability growth
Choice of Prior Distribution on Failure Intensity

- Assumes failure mode failure intensities realized from common Gamma($\alpha$, $\beta$) distribution
- Gamma Follows “vital-few, trivial-many” construct
- Can use Empirical Bayes to estimate parameters

$$p(\lambda) = \frac{\lambda^{\alpha-1}}{\Gamma(\alpha)\beta^\alpha} \exp\left(-\frac{1}{\beta} \lambda\right)$$

Provides inherent connection between failure modes

\[
p(\lambda_i | n_i) = \frac{\lambda_i^{\alpha+n_i-1}}{\Gamma(\alpha+n_i)\left[\frac{1}{\beta} + v_i + (1-d_i)(T_{DT} - v_i)\right]} \exp\left[-\lambda_i\left(\frac{1}{\beta} + v_i + (1-d_i)(T_{DT} - v_i)\right)\right]
\]
System Level Result

- System level estimate:
  - Summing over $m$ total failure modes
  - Take limit as $m$ becomes large
  - $n$ number of failures for each of the $m$ observed failure modes, i.

- For $m$ observed modes, system level mean is

$$E[\lambda_s] = \sum_{i=1}^{m} \left( \frac{(1-d_i)n_i}{1 + \frac{\lambda_B}{\beta} + (1-d_i)(T-v_i)} \right) + \left( \frac{\lambda_B}{1 + \beta T} \right)$$

where $\lambda_B = m\alpha\beta \equiv$ prior mean

Estimate includes contribution from unobserved modes
System Level Posterior Distribution

- Posterior can be simulated to determine approximate distribution
  - Exactly Gamma if corrective actions are delayed
  - Gamma approximation reasonable for arbitrary corrective action strategies

\[ \mu = E[\lambda_s] \]
\[ \beta' = \frac{Var[\lambda_s]}{E[\lambda_s]} \]
\[ \lambda_s \sim \text{Gamma} \left( \alpha' = \frac{\mu}{\beta'}, \beta' \right) \]

Can use mean and variance to develop system level posterior
Incorporating Operational Data

• Generally have increased failure intensity in operational environment
  – DT conditions more benign, human factors, etc.
• Define MTBF as reciprocal of mode failure intensity $\lambda$
• Assume $100\gamma\%$ decrease (degradation factor) in instantaneous MTBF ($1/\lambda$) such that
  \[ \lambda_{DT} = (1 - \gamma) \lambda_{OT} \]
• Transformed prior found using properties of Gamma distribution
  \[ \lambda_{OT} | \gamma = \frac{\lambda_{DT}}{(1 - \gamma)} \sim Gamma \left[ \alpha', \frac{\beta'}{(1 - \gamma)} \right] \]

Scaled prior accounts for reliability degradation
Marginal Posterior Distribution

- Assume $n_2$ failures in operational test length $T_2$
- Treats degradation factor $\gamma$ as nuisance parameter
- Marginal posterior development
  - Compute joint posterior
  - Compute marginal distribution by integrating over nuisance parameter
  \[
  l(t_{OT,1}, t_{OT,2}, \ldots, t_{OT,n_{OT}}, n_{OT} | \lambda_{OT}) = \lambda_{OT}^{n_{OT}} \exp(-\lambda_{OT} T_{OT})
  \]
- Use Beta prior distribution for $\gamma$

Marginal posterior probabilistically accounts for degradation
OT Posterior Assessment

- Posterior mean is scaled mean for Gamma distribution

\[
E[\lambda_s | n] = \left( \frac{\mu + n_2}{\beta' + n_2 + T_2} \right) \frac{2_{F1} \left[ \frac{\mu}{\beta'} + n_2 + 1, a, a + b + \frac{\mu}{\beta'}, \frac{1}{\beta' + T_2} \right]}{\frac{2_{F1} \left[ \frac{1}{\beta'} + n_2, a, a + b + \frac{\mu}{\beta'}, \frac{1}{\beta' + T_2} \right]}{1}}
\]

Standard posterior mean for Gamma

where \( \mu, \beta' \) defined as on slide 9

Ratio of Hypergeometric functions accounts for DT/OT degradation
System Level OT Posterior Distribution

- Exact system-level posterior can be simulated using Markov Chain Monte Carlo methods
- Posterior well approximated with Gamma distribution
- Use approximate Gamma to develop probability intervals

Can use mean and variance to develop system level posterior
## Performance Comparisons

A simulation was developed to examine relative error between model estimate and “true” value. The Bayesian approach performs better than current methods.

### Case Comparison

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial DT MTBF</th>
<th>DT Length</th>
<th>OT Length</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>1000</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
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<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>2000</td>
<td>500</td>
</tr>
</tbody>
</table>

### Mean Relative Error

![Mean Relative Error Graph]

- **Classical**
- **Bayes**

Full error distribution comparisons available in paper.

Bayesian approach performs better than current methods.
Conclusions/Future Work

• Use of reliability data from developmental testing provides additional information that increases performance of overall reliability estimate

• Bayesian probabilistic approach provides flexibility
  – Can utilize multiple information sources
  – Can include additional sources of uncertainty

• Current/future efforts include:
  – Modeling uncertainty on FEF values
  – Developing prior information from additional data sources
  – Analogous results for discrete systems